

Origin of Mass - Horizons Expanding from the Nambu's Theory

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The visible (Matter) has no difference from the invisible (Vacuum), the invisible has no difference from the visible. The visible is nothing but the invisible, the invisible is nothing but the visible.

- Heart Sutra (translation by KY)

Origin of mass may be strong dynamics of matter in the vacuum. Since the initial proposal of Nambu for the origin of the nucleon mass, the dynamical symmetry breaking in the *strongly coupled underlying theories* has been expanding the horizons in the context of the modern version of the origin of mass beyond the Standard Model (SM).

The Nambu-Jona-Lasinio (NJL) model is a typical strong coupling theory with the *non-zero critical coupling and a large anomalous dimension* $\gamma_m = 2$, in sharp contrast to its precedent model, the Bardeen-Cooper-Schrieffer theory for the superconductor. The non-zero critical coupling is also hidden in the asymptotically free gauge theories including QCD and walking technicolor: it reveals itself in the chiral symmetry restoration where the coupling cannot grow above the “hidden” critical coupling in the infrared region (infrared conformality).

As is well known, the NJL model can be cast into the SM Higgs Lagrangian. We show that the *SM Higgs Lagrangian is simply rewritten into a form of the (approximately) scale-invariant nonlinear sigma model*, with both the chiral symmetry and scale symmetry realized nonlinearly, with the SM Higgs being nothing but the (pseudo-) dilaton. The SM Higgs Lagrangian is *further gauge equivalent to the scale-invariant Hidden Local Symmetry (HLS) Lagrangian*, s-HLS, having spin 1 bosons hidden in the SM.

As the simplest possible underlying theory for the SM Higgs Lagrangian we first discuss the top quark condensate (“top-mode SM”) based on the (scale-invariant) NJL model with only top (plus possibly bottom) coupling larger than the critical coupling, where the top-mode dilaton is the 125 GeV Higgs and the HLS gauge boson (“top-mode rho meson”) (and the top-mode axion) may be detected at LHC.

We then discuss the walking technicolor having near infrared conformality and large anomalous dimension $\gamma_m \simeq 1$. Its effective theory is the s-HLS model precisely the same as the SM Higgs Lagrangian (with larger chiral symmetry), where the 125 GeV Higgs is successfully identified with the techidilaton. The 2 TeV diboson and 750 GeV diphoton excesses at LHC are identified with the HLS technirho and the technipion, respectively.

1. Introduction

Professor Nambu made great achievements in so much inexhaustible depth and wideness, and thus it may be something like the picture of “The Blind Men and the Elephant” to talk about only a single aspect of his physics. But the subject I am going to talk about is not just one of them, but probably his most influential one. In fact the 2018 Nobel Prize

announcement is “for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics” [1]. His spontaneous symmetry breaking (SSB) [2] was the theory for the origin of mass of the nucleon, then an elementary particle, generated dynamically from nothing (the vacuum) through the nucleon-antinucleon pair condensate. Although the nucleon is no longer an elementary particle, the essence of his mechanism to generate mass of composite particles, hadrons including the nucleon, as well as the near masslessness of the composite pion, is now realized through the quark-antiquark condensate in the underlying theory QCD. This mass constitutes 99 % of mass of the nucleon, namely of the ordinary matter made out of the atoms, and thus the Nambu’s theory already accounted for the origin of the dominant part of the mass of the visible world.

The problem of the origin of mass in the modern particle physics is only for the rest 1% of the mass of the matter, the elementary particles of the Standard Model (SM), which is attributed to the Higgs boson whose origin is still mysterious. I will discuss that this 1% may also be explained by the dynamical symmetry breaking in some underlying theory, similarly to the Nambu’s theory.

Origin of Mass of all the SM particles is the Higgs VEV $v = \sqrt{\frac{-\mu_0^2}{\lambda}} = 246 \text{ GeV}$ or the Higgs mass $M_\phi^2 = 2\lambda v^2 = -2\mu_0^2$ read from the SM Higgs Lagrangian:

$$\mathcal{L}_{\text{Higgs}} = |\partial_\mu h|^2 - \mu_0^2 |h|^2 - \lambda |h|^4 \quad (1)$$

$$= \frac{1}{2} [(\partial_\mu \hat{\sigma})^2 + (\partial_\mu \hat{\pi}_a)^2] - \frac{1}{2} \mu_0^2 [\hat{\sigma}^2 + \hat{\pi}_a^2] - \frac{\lambda}{4} [\hat{\sigma}^2 + \hat{\pi}_a^2]^2 \quad (2)$$

$$= \frac{1}{2} \text{tr} \left(\partial_\mu M \partial^\mu M^\dagger \right) - \left[\frac{\mu_0^2}{2} \text{tr} \left(M M^\dagger \right) + \frac{\lambda}{4} \left(\text{tr} \left(M M^\dagger \right) \right)^2 \right], \quad (3)$$

where we have rewritten the conventional form in Eq.(1) into the Gell-Mann-Levy (GL) $SU(2)_L \times SU(2)_R$ linear sigma model [3] in Eq.(2) through

$$h = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i\hat{\pi}_1 + \hat{\pi}_2 \\ \hat{\sigma} - i\hat{\pi}_3 \end{pmatrix}, \quad (4)$$

and further into Eq.(3) with the 2×2 matrix M

$$M = (i\tau_2 h^*, h) = \frac{1}{\sqrt{2}} (\hat{\sigma} \cdot 1_{2 \times 2} + 2i\hat{\pi}) \quad \left(\hat{\pi} \equiv \hat{\pi}_a \frac{\tau_a}{2} \right), \quad (5)$$

which transforms under $G = SU(2)_L \times SU(2)_R$ as:

$$M \rightarrow g_L M g_R^\dagger, \quad (g_{R,L} \in SU(2)_{R,L}). \quad (6)$$

Then the origin of mass is attributed to the mysterious input mass parameter of the *tachyons*, $\hat{\pi}$ and $\hat{\sigma}$ (not physical particles), with the mass μ_0 such that

$$\mu_0^2 < 0 \quad (7)$$

as a *free parameter*. But why the tachyon? How is the tachyon mass determined? SM cannot answer to these questions, even though the Higgs boson has been discovered with the mass near 125 GeV.

Historically, the GL linear sigma model in the form of Eq.(2) as the prototype of the Higgs Lagrangian Eq.(1) was proposed for phenomenologically describing the pion (as well as the nucleon) *without concept of the SSB*, while the Nambu-Jona-Lasinio (NJL) model [2]

explained the same property at the deeper level in terms of the dynamical symmetry breaking due to the vacuum property. Here we should recall that *the SSB was born as a dynamical symmetry breaking (DSB)*, where the tachyons are in fact generated as composites of the dynamical consequence of the strong dynamics, but not ad hoc inputs as in the GL theory. Actually, the GL theory is now regarded as an effective theory (macroscopic theory) for the NJL model as a microscopic theory, as is the Ginzburg-Landau (GL) theory for the Bardeen-Cooper-Schrieffer (BCS) theory [4] for the superconductor. So history may repeat itself.

I first discuss that although his idea was motivated by the BCS theory, it was not just a copy of it but essentially new in the most important aspect, namely it created a new dynamics, although based on the same kind of four-fermion interaction: the NJL dynamics is the *strong coupling theory having non-zero critical coupling* to separate the SSB phase with $\mu_0^2 < 0$ (above the critical coupling) from the non-SSB phase with $\mu_0^2 > 0$ (below the critical coupling). It is in sharp contrast to the BCS theory which is a *weak coupling theory having the zero critical coupling*, always in the SSB phase $\mu_0^2 < 0$ even for infinitesimal (attractive) coupling, due to the Fermi surface. The Fermi surface reduces the effective dimensions by 2 so as to make the theory in effectively 1 + 1 dimensions like the Thirring model and/or Gross-Neveu model.

The non-zero critical coupling is also hidden in the asymptotically free gauge theories including the QCD: it reveals itself in the chiral symmetry restoration when the system is in the extreme condition such as the high temperature, high density, and large number of light fermions, where the coupling cannot grow above the hidden critical coupling in the infrared region as strong enough to form the fermion-antifermion condensate. The existence of the non-zero critical coupling in the gauge theory was first recognized by Maskawa-Nakajima [5] in the ladder Schwinger-Dyson (SD) equation, a gauge theory analogue of the NJL gap equation. The solution of the ladder SD equation in the weak coupling region (existing even for the infinitesimal coupling) [6] disappears at zero fermion bare mass at finite cutoff, which is actually the explicit chiral symmetry breaking solution vanishing when the cutoff is removed.¹ Although the asymptotically-free gauge theory like QCD has no explicit critical coupling to divide the SSB phase from non-SSB phase (having only a single phase of SSB), the running coupling always becomes strong in the infrared region where the coupling exceeds a hidden critical coupling to trigger the SSB having the condensate of order of the scale of this mass region [7].

The main purpose of this article is to describe the expanding horizon of such a strong coupling dynamics characterized by the non-zero critical coupling initiated by Professor Nambu in view of the modern version of the origin of mass, namely the composite Higgs models having large anomalous dimension. First, the (weakly gauged) strong coupling four-fermion models like the top quark condensate model [8–10], where only the top quark has the strong

¹ At TV interview after announcement of the Nobel prize together with Professor Nambu in 2008, Toshihide Maskawa confessed that the paper he most studied was the Nambu's paper on SSB, "I exhausted it". He in fact discovered the non-zero critical coupling for SSB in the gauge theory [5] not just in the NJL four-fermion model. At that time I was a graduate student at Kyoto University to hear it first hand and have been influenced by this work, strong coupling gauge theory (SCGT) with non-zero criticality, ever since. See Nagoya SCGT workshops, <http://www.kmi.nagoya-u.ac.jp/workshop/SCGT15/>

coupling above the criticality (anomalous dimension $\gamma_m \simeq 2$ [11]) so as to be responsible for the electroweak symmetry breaking [8]. Second, the gauge theories such as the walking technicolor based on the near conformal gauge theory just above the criticality having anomalous dimension $\gamma_m \simeq 1$ and a composite dilaton (technidilaton) as the composite Higgs [12, 13]. The technidilaton in the walking technicolor has been shown to be consistent with the 125 GeV Higgs at the present LHC experimental data [14–16].

Before discussing possible underlying theory for SM, I show that the SM Higgs Lagrangian itself already has some hints for the theory beyond the SM. It was shown [17] that the SM Higgs Lagrangian itself possesses nonlinearly realized “hidden” symmetries (scale symmetry and Hidden Local Symmetry (HLS) [18–20], both spontaneously broken), in addition to the well-known symmetry, nonlinearly realized global $SU(2)_L \times SU(2)_R$ chiral symmetry (also spontaneously broken) to be gauged by the electroweak symmetry. It is in fact straightforward to show [17] that the SM Higgs Lagrangian is cast into the scale-invariant nonlinear chiral Lagrangian [14], and then further shown to be gauge equivalent to the scale-invariant HLS (s-HLS) Lagrangian [21]. The SM Higgs is nothing but a (pseudo-) dilaton! [17] *This is the very nature of the SM Higgs Lagrangian, quite independent of details of the possible underlying theory as the UV completion.* Also the HLS can naturally accommodate the vector bosons, analogues of the rho mesons in the QCD, into the SM (“SM rho meson”) [22]: It would be the simplest extension of the SM to account for the 2 TeV diboson events at LHC [23].

Then I elaborate [24] on the well-known fact that the NJL model can be regarded as the microscopic theory (underlying theory or ultraviolet (UV) completion) for the SM Higgs Lagrangian, or the GL linear sigma model, as the macroscopic theory (effective theory) at composite level. With the coupling larger than the non-zero critical coupling, the NJL model equivalent to the SM Higgs has also the nonlinearly realized hidden (approximate) scale symmetry for the SM Higgs as a composite pseudo-dilaton (“NJL dilaton”), together with the HLS for the dormant composite spin 1 boson (“NJL rho meson”) as a possible candidate for the LHC diboson events [23]. Although both are trivial theories having no interaction in the infinite cutoff limit (Gaussian fixed point), I will discuss possible way out, one [25] being the gauged NJL model in combination with the walking gauge theory, another [24] the recently suggested different way of the continuum limit where the composite Higgs becomes massless (up to the trace anomaly) as the pseudo-dilaton in the same sense as the SM Higgs.

The simplest possibility for such a composite model would be the top quark condensate model (“top-mode SM”) [8–10], where crucial is the non-zero criticality [8]: only top (may also bottom) has the coupling larger than the non-zero critical coupling to acquire the dynamical mass due to SSB. Near the scale-invariant limit, the top-mode dilaton may be the 125 GeV Higgs, and the HLS gauge boson (“top-mode rho meson”) may be identified with the recent 2 TeV diboson excess (and the top-mode axion, $\bar{b}b$ bound state, may be identified with the 750 GeV diphoton excess at LHC [26] which was reported after this symposium).

We then discuss the walking technicolor proposed based on the SSB solution of the ladder SD equation to have a large anomalous dimension $\gamma_m = 1$ and technidilaton as a pseudo-Nambu-Goldstone (NG) boson of the approximate scale symmetry [12, 13]. Such a scale-symmetric walking gauge theory may be realized when flavor number N_F of massless fermions is large in the asymptotically free gauge theory (“large N_F QCD”) [27, 28], with $N_F (\gg 2)$ slightly smaller than that having an infrared fixed point (conformal window) where the coupling in the infrared region is almost constant and below the critical coupling so that the SSB does not take place. The effective theory of the walking technicolor is the s-HLS Lagrangian with a larger chiral symmetry $SU(N_F)_L \times SU(N_F)_R$, with typically $N_F = 8$ (one-family model), precisely the same type of the s-HLS as in the case for the SM Higgs Lagrangian with $SU(2)_L \times SU(2)_R$. The technidilaton as a composite Higgs has been shown [14–16] to be consistent with the present LHC 125 GeV Higgs, and the HLS vector mesons (walking technirhos) have also been shown [29] to be consistent with the LHC diboson events [23]. (We also showed [30] that one of the technipions can be identified consistently with the 750 GeV diphoton events at LHC [26] reported after the symposium).

Several theoretical issues are discussed such as the recent lattice studies of the walking theories, as well as the ladder, and the renormalizability of the gauged-NJL model and the conformal phase transition, etc.

2. NJL the Strong Dynamics vs. BCS the Weak Dynamics

It is widely believed that the NJL model is a copy of the BCS. Here I emphasize that they are essentially different dynamics, NJL as the strong coupling with critical coupling no-zero, while the BCS as a weak coupling with the critical coupling zero. The difference comes from the Fermi surface in BCS which reduces the effective phase space from 3+1 to 1+1, while the NJL case is in the free space of full 3+1 dimensions. The attractive forces are more efficient in smaller phase space.

Let us start with the $SU(2)_L \times SU(2)_R$ NJL model [2] for N_C 2-flavored Dirac fermions ψ :

$$\mathcal{L}_{\text{NJL}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \frac{G}{2} [(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau^a \psi)^2] . \quad (8)$$

When the fermion-antifermion condensate in the vacuum takes place, $\langle 0 | \bar{\psi}\psi | 0 \rangle \neq 0$, it reads

$$\mathcal{L}_{\text{NJL}} - \bar{\psi} i \gamma^\mu \partial_\mu \psi = G \langle \bar{\psi}\psi \rangle \bar{\psi}\psi + \dots = -m_F \bar{\psi}\psi + \dots . \quad (9)$$

At the $1/N_C$ leading order this yields the self-consistent NJL gap equation for the dynamical mass m_F of ψ :

$$m_F = -G \langle \bar{\psi}\psi \rangle = G \text{Tr}[S_F(p)] = G \cdot 4N_C \int \frac{d^4 p}{i(2\pi)^4} \frac{m_F}{m_F^2 - p^2} \quad (10)$$

which has an SSB solution $m_F \neq 0$:

$$\frac{1}{G} - \frac{1}{G_{\text{cr}}} = \frac{\Lambda^2}{4\pi^2} \left(\frac{1}{g} - \frac{1}{g_{\text{cr}}} \right) = -\frac{1}{4\pi^2} N_C m_F^2 \ln \left(\frac{\Lambda^2}{m_F^2} \right) < 0, \quad (11)$$

only for the strong coupling

$$G > G_{\text{cr}} = \frac{4\pi^2}{N_C \Lambda^2} \neq 0 \quad (g \equiv \frac{G \Lambda^2}{4\pi^2} > g_{\text{cr}} = \frac{1}{N_c} \neq 0). \quad (12)$$

We shall later discuss that this in fact corresponds to the tachyon mass $\mu_0^2 < 0$ in Eq.(2): $\mu_0^2 = (\frac{1}{G} - \frac{1}{G_{\text{cr}}}) \cdot Z_\phi^{-1} = -2m_F^2 < 0$, where $Z_\phi = \frac{N_C}{8\pi^2} \ln \frac{\Lambda^2}{m_F^2}$. The (composite) tachyon has

been induced dynamically by the $-1/G_{\text{cr}}$ term due to the loop effects in the large N_C limit. Of course the tachyon is not a physical particle, which simply implies instability of the trivial vacuum with $m_F = 0$ (no SSB). For the weak coupling $G < G_{\text{cr}}$ there exists only the non SSB solution $m_F \equiv 0$ where no tachyon exists.

Note that “strong coupling” as defined by non-zero critical coupling does not necessarily mean numerically strong, particularly in the large N_C limit $g_{\text{cr}} = 1/N_C \ll 1$. However, the attractive forces in the condensate channel are not from a single fermion but actually from sum of all the N_c fermions coherently, which ends up with really strong $N_C g_{\text{cr}} = \mathcal{O}(1)$. [As we discuss later, this also applies to the strong coupling gauge theory where $N_C \alpha_{\text{cr}} = \mathcal{O}(1)$, while the gauge coupling criticality itself $\alpha_{\text{cr}} \sim 1/N_C \ll 1$ is negligibly small (but non-zero) in the large N_C limit].

In contrast to the non-zero critical coupling of the NJL model, the BCS theory for the superconductor has the zero critical coupling (“weak coupling theory”) due to the electron Fermi surface $E_F = \frac{p_F^2}{2m_e}$ (m_e : electron mass in the free space), which affects the fermion-fermion condensate $\langle \psi\psi \rangle \neq 0$ (dynamical Majorana mass Δ as the gap) instead of fermion-antifermion condensate $\langle \bar{\psi}\psi \rangle \neq 0$. The essence can be read from the effective dimension of the momentum in the integral of the gap equation of the BCS $\int \frac{d^4 p}{i(2\pi)^4} = \int \frac{d^3 p}{(2\pi)^3} \frac{d\omega}{i(2\pi)}$:

$$\int \frac{d^3 p}{(2\pi)^3} = \int \frac{(4\pi p^2) dp}{(2\pi)^3} \Rightarrow 4\pi p_F^2 \int_{|E(p)-E_F| < \omega_D/2} \frac{dp}{(2\pi)^3} = \frac{N}{2} \int_{E_F - \omega_D/2}^{E_F + \omega_D/2} dE(p), \quad (13)$$

where $E(p) \equiv \frac{p^2}{2m_e}$ and $N \equiv \frac{m_e p_F}{\pi^2} = \text{constant}$: The 3-dimensional electron momentum \vec{p} is confined to a one-dimensional direction normal to the Fermi surface $E_F = \frac{p_F^2}{2m_e}$ in the narrow energy shell bounded by the Debye energy ω_D (cutoff). After integral $\int d\omega$, with the fermion propagator $S_F(p)^{(\text{Majorana})} = \mathcal{F.F.} \langle T(\psi(x)\psi(0)) \rangle = \Delta/(\omega^2 - |\Delta|^2 - E(p)^2)$ (instead of $\mathcal{F.F.} \langle T(\bar{\psi}(x)\psi(0)) \rangle$), the BCS gap equation corresponding to Eq.(10) (Majorana mass without factor 4) reads

$$|\Delta| = G \frac{N}{2} \int_0^{E_F + \omega_D/2} dE(p) \frac{|\Delta|}{\sqrt{|\Delta|^2 + E(p)^2}} \sim |\Delta| \left[\frac{NG}{2} \ln \frac{|\Delta|}{\omega_D} \right]. \quad (14)$$

Then the SSB solution with $|\Delta| \neq 0$ exists even for infinitesimal coupling $1 \gg NG > NG_{\text{crit}} = 0$:

$$|\Delta| \sim \omega_D \exp\left(-\frac{2}{NG}\right), \quad (1 \gg NG > NG_{\text{cr}} = 0), \quad (15)$$

in contrast to the SSB solution in NJL model in Eq.(11) with $N_C g > N_C g_{\text{cr}} = \mathcal{O}(1)$ in Eq.(12).

The result is intuitively obvious: The fermion pair in the one dimensional space is “bound” even for infinitesimal coupling, since there is no way to escape from each other, while that in the higher dimensional space can freely move from each other and hence needs strong attractive forces to bind it together. This is the effective dimensional reduction. The situation that the lower dimensional theory lowers the critical coupling can be viewed explicitly by the $D(1+1 < D < 3+1)$ dimensional four-fermion theory, the Gross-Neveu model, with D changed continuously [31]. The gap equation is simply changed as $\int \frac{d^4 p}{i(2\pi)^4} \Rightarrow \int \frac{d^D p}{i(2\pi)^D}$ in Eq.

(10). Similarly to Eq.(11), the SSB solution exists:[31]

$$\frac{1}{g} - \frac{1}{g_{\text{cr}}} = -\frac{N_C \xi_D}{2 - \frac{D}{2}} \cdot \left(\frac{m_F}{\Lambda}\right)^{D-2} < 0, \quad (16)$$

only for the strong coupling;²

$$g > g_{\text{cr}} = \frac{1}{N_C} \left(\frac{D}{2} - 1\right) \rightarrow 0 \quad (D \rightarrow 2), \quad (17)$$

where $g \equiv G\Lambda^{D-2} \frac{2^{D/2}}{(4\pi)^{D/2}\Gamma(D/2)}$ and $\xi_D = B(\frac{D}{2} - 1, 3 - \frac{D}{2}) \rightarrow \frac{1}{D/2-1} = \frac{1}{N_C g_{\text{cr}}}$ for $D \rightarrow 2$ ($\rightarrow 1$ for $D \rightarrow 4$)³. The critical coupling g_{cr} indeed decreases as D does to vanish at $D = 2$. This yields for $D \rightarrow 2$ the well-known result:

$$m_F = \Lambda \exp\left(-\frac{1}{2N_C g}\right), \quad (N_C g > N_C g_{\text{cr}} = 0), \quad (18)$$

which is of the same form as Eq.(15).

Thus the BCS dynamics in some sense is similar to the $D = 1 + 1$ four-fermion theories such as the Thirring model and the Gross-Neveu model. There is a caveat [33], however: the genuine $D = 1 + 1$ dimensional theory is not actually in the SSB phase in accord with the Merwin-Wagner-Coleman theorem, although it has a massless bound state and a massive fermion with mass of the form of Eq.(18) in the large N_C limit, similarly to the SSB phase. However, the absence of the NG boson and lack of SSB does not apply to the BCS theory in contrast to the Thirring model and Gross-Neveu model, since the BCS theory is not a genuine $1 + 1$ dimensional model but rather a brane model: only fermions (not anti-fermions) are confined to the $1 + 1$ -brane, the Fermi surface, a consequence of the Fermi statistics, while the fermion-fermion pair composite NG boson as a boson lives freely from the Fermi surface in the full $3 + 1$ dimensional bulk, and hence SSB and NG boson do exist, in accord with the superfluidity and superconductor.

To summarize the Nambu's approach to the origin of mass, the theory having intrinsic mass scale $\Lambda \sim G^{-1/2}$ may or may not produce the particle mass m_F , depending on the coupling strength: the strong coupling dynamics for $G > G_{\text{cr}} \neq 0$ creates the composite tachyon with negative mass² $\mu_0^2 \sim 1/G - 1/G_{\text{cr}} = -\frac{N_C}{4\pi^2} m_F^2 \ln \frac{\Lambda^2}{m_F^2} < 0$ in such a way that the particle mass m_F is generated from the intrinsic mass scale Λ . By fine tuning the strong coupling $G(> G_{\text{cr}} \neq 0)$ as $G \simeq G_{\text{cr}}$, we can arrange a big hierarchy $m_F \ll \Lambda$ (near chiral symmetry restoration). On the other hand, for the weak coupling $G < G_{\text{cr}}$ there exists no particle mass $m_F \equiv 0$, although the theory has an intrinsic mass scale Λ . This is an essential difference from the BCS theory which has a zero critical coupling, producing always a non-zero gap $\Delta \neq 0$ even for the infinitesimal coupling.

² In $D > 4$ dimensions, the form $g_{\text{cr}} = \frac{1}{N_C} (\frac{D}{2} - 1)$ remains the same, in accord with the above intuitive picture for the required binding force strength depending on the phase volume, while the gap equation takes a similar but different form: $1/g - 1/g_{\text{cr}} = -\frac{N_C}{D/2-2} \cdot (\frac{m_F}{\Lambda})^2$ [49].

³ If we take the $D \rightarrow 4$ limit, on the other hand, the gap equation is reduced to Eq.(11) except for the logarithmic factor. This log factor is a crucial difference between the $2 < D < 4$ and the $D = 4$ four-fermion theories. As we discuss later, the former is renormalizable in $1/N_C$ expansion having the nontrivial fixed point at $g = g_{\text{cr}}$ in the beta function, $\beta(g) = -\frac{D-2}{g_{\text{cr}}} g(g - g_{\text{cr}})$ [31], while the latter is not, a trivial theory, with the beta function having the Gaussian fixed point at $g = g_{\text{cr}}$. ($g = 0$ is an infrared fixed point defining the infrared free theory, with $g < 0$ being the repulsive forces.)

As discussed later, this non-BCS phase structure of the NJL dynamics was in fact the original motivation of the top quark condensate model of Ref. [8], where the top quark having a coupling larger than the critical coupling is discriminated from others having those smaller than the critical coupling, so that only the top has mass of order of weak scale in such a way as to produce only three NG bosons responsible for the electroweak symmetry breaking. This is in contrast to the “bootstrap symmetry breaking” [9] which is based on the BCS dynamics without the notion of the non-zero criticality.

3. Strong Coupling Gauge Theories for the Origin of Mass

The dynamical mass of the fermion m_F picks up the intrinsic scale Λ (cutoff) which regularizes the theory and brings the explicit breaking of the scale symmetry corresponding to the trace anomaly in the renormalized quantum theory. In the asymptotically free gauge theory Λ can be identified with the renormalization-group invariant intrinsic scale such as Λ_{QCD} induced by the perturbative trace anomaly, as we discuss later.

There also exists a non-zero critical coupling for SSB in the gauge theory with massless fermion, as first noted [5] in the ladder SD equation, with non-running coupling $\alpha(\mu^2) \equiv \alpha = g^2/(4\pi)$ in the Landau gauge, a straightforward extension of the NJL gap equation Eq. (10), this time for the fermion mass function $\Sigma(-p^2)$ instead of the constant mass m_F (For details see e.g., Ref.[16]):

$$S_F^{-1}(p) = S^{-1}(p) + \int \frac{d^4k}{(2\pi)^4} C_2 g^2 D_{\mu\nu}(p-k) \gamma^\mu S_F(k) \gamma^\nu, \quad (19)$$

where $iS_F^{-1}(p) = Z^{-1}(-p^2)(\not{p} - \Sigma(-p^2))$ and $iS^{-1}(p) = (\not{p} - m_0)$ are the full and bare fermion inverse propagators, respectively, and $iD_{\mu\nu}(p) = (g_{\mu\nu} - p_\mu p_\nu/p^2)/p^2$ the bare gauge boson propagator in the Landau gauge, and C_2 is the quadratic Casimir of the fermion of the gauge theory, with $C_2 = (N_C^2 - 1)/(2N_C)$ for the fundamental representation in $SU(N_C)$. After the angular integration, the ladder SD equation in Landau gauge for $\Sigma(x \equiv -p^2)$ reads:

$$\Sigma(x) = m_0 + \frac{3C_2}{4\pi} \alpha \int^{\Lambda^2} dy \left[\frac{\theta(x-y)}{x} + \frac{\theta(y-x)}{y} \right] \frac{y\Sigma(y)}{y + \Sigma^2(y)}, \quad (Z^{-1}(x) \equiv 1). \quad (20)$$

This form is reduced back to the form of the NJL gap equation with $\Sigma(x) \equiv m_F$, Eq.(10), if the kernel is local: $\theta(x-y)/x + \theta(y-x)/y \rightarrow 1/\Lambda^2$, such as in the case of the massive gauge boson, $iD_{\mu\nu} \sim g_{\mu\nu}/\Lambda^2$ (See also Eq.(70)).

Eq.(20) is converted into a differential equation plus IR and UV boundary conditions [34]:

$$(x\Sigma(x))'' + \alpha \frac{3C_2}{4\pi} \frac{\Sigma(x)}{x + \Sigma^2(x)} = 0, \quad (21)$$

$$\lim_{x \rightarrow 0} x^2 \Sigma'(x) = 0, \quad (22)$$

$$(x\Sigma(x))'|_{x=\Lambda^2} = m_0. \quad (23)$$

The asymptotic solution of Eq.(21) at $x \gg \Sigma^2(x)$ takes the form $\Sigma(x) \sim m_F(x/m_F^2)^a$, with a conventional normalization $\Sigma(x = m_F^2) = m_F$, which is plugged back into the equation to yield $(a+1)a + \alpha(3C_2)/(4\pi) = 0$, i.e., $a = (-1 \pm \sqrt{1 - 3C_2\alpha/\pi})/2$.

For $\alpha < \frac{\pi}{3C_2} \equiv \alpha_{\text{cr}}$, either solution, dominant ($a = (-1 + \omega)/2$) or non-dominant ($a = (-1 - \omega)/2$), has a power behavior, which does not satisfy the UV boundary condition Eq.(23) for the chiral limit $m_0 = 0$, where $\omega \equiv \sqrt{1 - \alpha/\alpha_{\text{cr}}}$. The solution exists only at

the presence of the explicit breaking m_0 , namely the explicit breaking solution with the renormalized mass $m_F = m_R = Z_m^{-1} m_0$, which yields the anomalous dimension γ_m in the unbroken phase [35]:

$$m_0 = m_R \left(\frac{\Lambda}{m_R} \right)^{-1+\omega}, \quad \gamma_m = \Lambda \frac{\partial \ln Z_m^{-1}}{\partial \Lambda} = 1 - \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}} < 1 \quad \left(\alpha < \alpha_{\text{cr}} = \frac{\pi}{3C_2} \right). \quad (24)$$

The result is written in terms of the one-loop anomalous dimension $\gamma_m^{(\text{one-loop})} = 3C_2\alpha/(2\pi)$ as $\gamma_m = 1 - \sqrt{1 - 2\gamma_m^{(\text{one-loop})}}$ which coincides with $\gamma_m^{(\text{one-loop})}$ for $\alpha/\alpha_{\text{cr}} \ll 1$:

On the other hand, the SSB solution does exist for

$$\alpha > \frac{\pi}{3C_2} = \alpha_{\text{cr}}, \quad (25)$$

where $a = (-1 \pm i\tilde{\omega})/2$ with $\tilde{\omega} \equiv \sqrt{\alpha/\alpha_{\text{cr}} - 1}$, and the solution is of the oscillating form $\Sigma(x) \sim \frac{m_F^2}{\sqrt{x}} \frac{1}{\tilde{\omega}} \sin(\tilde{\omega}[\ln(\sqrt{x}/m_F) + \delta])$, $\delta = \mathcal{O}(1)$, which satisfies the UV boundary condition as

$$0 = m_0 \sim \frac{m_F^2}{\Lambda \tilde{\omega}} \sin\left(\tilde{\omega} \ln\left(\frac{4\Lambda}{m_F}\right)\right), \quad (26)$$

for $\tilde{\omega} \ln\left(\frac{4\Lambda}{m_F}\right) = n\pi$ (numerically $e^\delta \simeq 4$). In the large N_C limit the critical coupling itself is numerically small, $\alpha_{\text{cr}} \sim 1/N_C \ll 1$, although the effective coupling in the condensate channel is $C_2\alpha_{\text{cr}} = \mathcal{O}(1)$ as was the case in the NJL coupling. This is the reason why the ladder approximation yields reasonable result.

The ground state solution is $n = 1$, which yields the dynamical mass m_F of the Berezinsky-Koterlitz-Thouless (BKT) form of essential-singularity (“Miransky scaling”) [37]:

$$m_F \simeq 4\Lambda \exp\left(-\frac{\pi}{\sqrt{\frac{\alpha}{\alpha_{\text{cr}}} - 1}}\right), \quad \left(\alpha > \alpha_{\text{cr}} = \frac{\pi}{3C_2} \neq 0\right), \quad (27)$$

$$= 0, \quad (\alpha < \alpha_{\text{cr}}). \quad (28)$$

This is compared with the NJL gap equation Eq. (11) and D-dimensional NJL Eq.(16), and also with the BCS Eq.(15) and 2-dimensional model Eq.(18). Again the large hierarchy

$$m_F \ll \Lambda \quad (\alpha/\alpha_{\text{cr}} - 1 \ll 1) \quad (29)$$

can be realized near criticality (near chiral symmetry restoration).

The essential-singularity scaling yields a peculiar phase transition, dubbed “conformal phase transition” [28], which is different from the typical 2nd order phase transition as the Ginzburg-Landau phase transition. While the order parameter such as m_F is continuously changed as $m_F \neq 0$ to $m_F = 0$ from $\alpha > \alpha_{\text{cr}}$ to $\alpha < \alpha_{\text{cr}}$, the spectrum changes discontinuously, since there is no light spectrum in $\alpha < \alpha_{\text{cr}}$ (conformal, unparticle) in contrast to the SSB phase where mass spectrum all goes to zero as $\alpha \rightarrow \alpha_{\text{cr}} + 0$. It reflects the fact that the essential singularity is not analytic at $\alpha = \alpha_{\text{cr}}$. The light spectrum is possible for $\alpha < \alpha_{\text{cr}}$ only when $m_0 \neq 0$ which violates the conformality. Thus all the mass spectrum M for the conformal phase $\alpha < \alpha_{\text{cr}}$ scales like the explicit breaking renormalized mass m_R , which is given by Eq.(24) as $M \sim m_R \sim m_0^{1/(1+\gamma_m)}$ [36], in conformity with the hyperscaling relation frequently used in the lattice analyses for the conformal signals.

Eq.(27) implies that the coupling α is a function of m_F/Λ with the nonperturbative beta function:

$$\beta^{(NP)}(\alpha) = \Lambda \frac{\partial \alpha(\Lambda)}{\partial \Lambda} = -\frac{2\pi^2 \alpha_{\text{cr}}}{\ln^3(\frac{4\Lambda}{m_F})} = -\frac{2\alpha_{\text{cr}}}{\pi} \left(\frac{\alpha}{\alpha_{\text{cr}}} - 1 \right)^{\frac{3}{2}}, \quad (30)$$

$$\alpha(\mu) = \alpha_{\text{cr}} \left[1 + \frac{\pi^2}{\ln^2(\frac{4\mu}{m_F})} \right], \quad (31)$$

with α_{cr} being now regarded as a nontrivial ultraviolet fixed point (approached much faster than the asymptotic freedom $\sim 1/\ln \mu$). The asymptotic form of the SSB solution is $\Sigma(x) \sim m_F^2/\sqrt{x}$ which is compared with the Operator Product Expansion $\Sigma(x) \sim \frac{m_F^3}{x} (\frac{x}{m_F^2})^{\gamma_m/2}$ to yield a large anomalous dimension: [12, 13]

$$\gamma_m = 1 \quad (\alpha > \alpha_{\text{cr}}). \quad (32)$$

This ladder result is the characteristic feature of the walking technicolor.

Due to this mass generation which breaks the scale symmetry spontaneously, the ladder scale symmetry is also broken explicitly producing the new nonperturbative trace anomaly besides the perturbative trace anomaly induced by the cutoff regularization Λ (See Ref. [16] and references cited therein):

$$\begin{aligned} \langle \partial_\mu D^\mu \rangle = \langle \theta_\mu^\mu \rangle^{(NP)} &\equiv \langle \theta_\mu^\mu \rangle^{(full)} - \langle \theta_\mu^\mu \rangle^{(perturbative)} = \frac{\beta^{(NP)}(\alpha)}{4\alpha} \langle G_{\mu\nu}^2 \rangle^{(NP)}, \\ &\simeq -N_F N_C \frac{4\xi^2}{\pi^4} m_F^4, \quad (\xi \simeq 1.1), \end{aligned} \quad (33)$$

where $\langle G_{\mu\nu}^2 \rangle^{(NP)} \equiv \langle G_{\mu\nu}^2 \rangle^{(full)} - \langle G_{\mu\nu}^2 \rangle^{(perturbative)}$ is the nonperturbative gluon condensate and N_F is a number of flavors of massless fermions (besides color N_C). Note that although $\frac{\beta^{(NP)}(\alpha(\mu))}{4\alpha(\mu)}$ and $\langle G_{\mu\nu}^2 \rangle^{(NP)}$ are depending on the renormalization point μ , the trace anomaly $\langle \theta_\mu^\mu \rangle^{(NP)}$ is not as it should be (the energy-momentum tensor $\theta_{\mu\nu}$ is a conserved current and is not renormalized), with both dependence being cancelled each other precisely. [16]

This ladder dynamics was the basis for the walking technicolor [12, 13] where the coupling is almost non-running $\alpha(\mu) \approx \alpha_{\text{cr}}$ for $m_F < \mu < \Lambda$ even after the SSB takes place to produce the nonperturbative running.

The non-zero critical coupling also exists in the asymptotically free gauge theory including the QCD in a more sophisticated way, in spite of no explicit non-zero critical coupling separating the SSB phase and the non-SSB phase, namely the QCD is in one phase always in the SSB similarly to the BCS. The theory is classically scale-invariant but actually has an intrinsic mass scale Λ_{QCD} due to the trace anomaly by the quantum effects (regulator). The intrinsic scale Λ_{QCD} is usually given by the one-loop beta function: $\Lambda_{\text{QCD}} = \mu e^{-1/(b_0 \alpha(\mu))} = \Lambda e^{-1/(b_0 \alpha(\Lambda))}$ with b_0 given in Eq.(35). Although this looks like the BCS mass generation in Eq.(15), Λ_{QCD} should not be confused with the mass generation m_F . The existence of the intrinsic scale Λ_{QCD} does not necessarily imply the SSB $m_F \neq 0$ as in the NJL model where the intrinsic scale $1/\sqrt{G}$ does not necessarily imply the mass m_F .

The QCD coupling $\alpha(\mu)$ runs depending on the renormalization scale μ in units of Λ_{QCD} to grow in the infrared region. The fermion mass m_F is dynamically generated due to the fermion-antifermion condensate which takes place in the infrared region $\mu < m_F$ where the coupling becomes strong as to exceed the “hidden” critical coupling of order 1: $N_C \alpha(\mu <$

$m_F) > N_C \alpha_{\text{cr}} = \mathcal{O}(1)$. In the usual QCD it so happens that $m_F = \mathcal{O}(\Lambda_{\text{QCD}})$. In a wider parameter space, however, we can see the cases $m_F = 0$ (chiral restoration) and $m_F \ll \Lambda_{\text{QCD}}$ (near chiral restoration), where the non-zero critical coupling is actually essential.

The “hidden” non-zero critical coupling become “visible” when the system is put in the medium with finite temperature T and density with baryon chemical potential μ_B , where the running coupling in the infrared region is no longer growing indefinitely and levels off at the relevant energy scale of order of T or μ_B . Then for T, μ_B such that $\alpha(T), \alpha(\mu_B) < \alpha_{\text{cr}}$ the SSB would not take place, namely the chiral symmetry restoration occurs as has been studied actively. In contrast to the disappearance of the fermion-antifermion condensate, the BCS dynamics for fermion-fermion condensate instead can be operative in the finite density even with the weakest coupling due to the Fermi surface, which is called color superconductor.

Here I discuss another case to visualize the non-zero critical coupling in the QCD-like vector-like $SU(N_C)$ gauge theory with $N_F (\gg N_C)$ massless technifermions, still in the asymptotically free theory $N_F < 11N_C/2$ with the running coupling vanishing in the ultra-violet region. This is the basis for the walking technicolor to be discussed later and I denote the intrinsic scale Λ_{QCD} as Λ_{TC} hereafter.

When one increases N_F , the vacuum polarization due to the virtual fermion-antifermion pairs (loop effects) increases the screening of the charges in the long distance (infrared energy region), which is operative opposite to the asymptotically free anti-screening effects of the gluon loops: in the ultraviolet region $\mu \gg \Lambda_{\text{TC}}$ the coupling is small and running is essentially one-loop dominated, while in the infrared region $\mu \ll \Lambda_{\text{TC}}$ where the coupling grows, the higher loop effects particularly by the fermion loop screening effects are getting dominant, which then balances the anti-screening effects to tend to make the coupling level off. Then the dynamical mass m_F such that $\alpha(\mu = m_F) \simeq \alpha_{\text{cr}}$ will be getting smaller, as we increase N_F/N_C :

$$\frac{m_F}{\Lambda_{\text{TC}}} \searrow \quad \text{for} \quad \frac{N_F}{N_C} \nearrow, \quad (34)$$

in contrast to the ordinary QCD with $m_F = \mathcal{O}(\Lambda_{\text{QCD}})$ for $N_F = N_C = 3$. It then eventually could realize at certain large $r \equiv N_F/N_C \gg 1$ an infrared fixed point $\alpha(\mu) < \alpha_* = \alpha(0) < \alpha_{\text{cr}}$, which implies that no SSB takes place and no bound states exist (“unparticle”), the phase called “conformal window”.⁴ The approximate scale symmetry is operative with almost nonrunning coupling in the infrared region $\mu \ll \Lambda_{\text{TC}}$, although it is violated explicitly by Λ_{TC} due to the trace anomaly in the ultraviolet region $\mu \gg \Lambda_{\text{TC}}$ where the coupling is running as in the usual asymptotically free theory.

The existence of the conformal window in fact can be seen explicitly at two-loop beta function, which is scheme-independent while higher loops are not: [38].

$$\begin{aligned} \beta^{(2\text{-loop})}(\alpha) &= -b_0 \alpha^2 - b_1 \alpha^3, \\ b_0 &= \frac{1}{6\pi} (11N_C - 2N_F), \quad b_1 = \frac{1}{24\pi^2} \left(34N_C^2 - 10N_C N_F - 3 \frac{N_C^2 - 1}{N_C} N_F \right), \\ \alpha_* &= \alpha_*(N_F, N_C) = -\frac{b_0}{b_1}, \end{aligned} \quad (35)$$

⁴ Here we are talking about the phase transition in the parameter N_F/N_C by changing the theory. It does not imply the existence of two phases in one theory with fixed N_F/N_C . See the discussions below and Fig. 1

where we have $\beta^{(2-loop)}(\alpha = \alpha_*) = 0$ by balancing the one-loop $-b_0\alpha^2$ (< 0 as far as asymptotically free, i.e., $N_F < 11N_C/2$) with the two-loop contributions $-b_1\alpha^3 > 0$ at the infrared limit $\mu = 0$, which is realized only when $b_1 < 0$ s.t. $N_F \gg N_C$ is satisfied. Note that $\alpha_* \searrow$ as $N_F/N_C \nearrow$ and $\alpha_* = \alpha_*(N_F, N_C)$ exists for $N_F^* < N_F < 11N_C/2$ ($N_F^* \simeq 8$ for $N_C = 3$).

In the context of large N_C limit, such a situation corresponds to the “anti-Veneziano limit” (in distinction to the original Veneziano limit with $N_F/N_C \ll 1$):[16]

$$N_C \rightarrow \infty \quad \text{and} \quad N_C \cdot \alpha = \text{fixed}, \quad \text{with} \quad r \equiv N_F/N_C = \text{fixed} \gg 1. \quad (36)$$

The anti-Veneziano limit in fact realizes a situation very close to the ladder approximation, with the $r = N_F/N_C$ behaving as a continuous parameter. Then the theory has two phases in the parameter space r : SSB phase for $r > r_{\text{cr}}$ such that $\alpha(\mu) > \alpha_{\text{cr}}$ and the non-SSB phase otherwise.

In the case $\alpha_* < \alpha_{\text{cr}}$, there in fact exists no SSB $m_F \equiv 0$ and no bound states (“unparticle”). The coupling is almost constant for all the infrared region $\mu < \Lambda_{\text{TC}}$ (infrared conformality), while it is running in the ultraviolet region $\mu > \Lambda_{\text{TC}}$ essentially as the one-loop running, in accord with the scale symmetry violation due to the perturbative trace anomaly.⁵

On the other hand, for $\alpha_* > \alpha_{\text{cr}}$ the SSB takes place with mass m_F generated similarly to the ladder SD result in Eq. (27) with α replaced by α_* [27],

$$m_F \sim \Lambda_{\text{TC}} \exp \left(-\frac{\pi}{\sqrt{\frac{\alpha_*}{\alpha_{\text{cr}}} - 1}} \right) \quad \left(\ll \Lambda_{\text{TC}} \quad \text{for} \quad \frac{N_F}{N_C} \quad \text{s.t.} \quad \frac{\alpha_*(N_F, N_C)}{\alpha_{\text{cr}}(N_C)} - 1 \ll 1 \right), \quad (37)$$

where the phase transition in the parameter space r has a characteristic essential singularity scaling (Miransky-BKT scaling), which takes the same type of the “conformal phase transition” as the ladder one [28].

Once m_F is generated, the scale symmetry is explicitly broken so as to yield the nonperturbative trace anomaly, Eq.(33), responsible for the nonperturbative running of the coupling, Eq.(31), and the would-be infrared fixed point α_* at two-loop is actually washed out. The resultant coupling would have a form with (quasi) ultraviolet fixed point α_{cr} similarly to Eq.(27) for $\alpha(\mu) > \alpha_{\text{cr}}$ ($\mu < \Lambda_{\text{TC}}$), while it still has a remnant of infrared fixed point (quasi fixed point) for $\alpha(\mu) < \alpha_{\text{cr}} \simeq \alpha_*$ ($\mu > \Lambda_{\text{TC}}$). Thus the theory is in one phase, which is not separated by α_{cr} . The beta function has no exact zero at α_{cr} and the coupling runs through α_{cr} continuously. See Fig.1.

To summarize the origin of mass in the strong coupling theories, the mass m_F originates from the intrinsic scale Λ (Lagrangian parameter G or the trace anomaly $\Lambda_{\text{QCD}}(\Lambda_{\text{TC}})$) through SSB which takes place only in the strong coupling phase with the coupling larger than the non-zero critical coupling. There is no mass generation $m_F \equiv 0$ in the weak coupling phase, even though the theory has intrinsic scale Λ .

⁵ For the region $\alpha(\mu) > \alpha_{\text{cr}} > \alpha_*$, there might exist SSB, in which case there might exist two phases separated by the ultraviolet fixed point at α_{cr} in the sense similar to the conjecture on the asymptotically non-free gauge theory such as the strong coupling QED for $\alpha(\mu) > \alpha_{\text{cr}}$, although such a fixed point may be a Gaussian fixed point (trivial theory) [39].

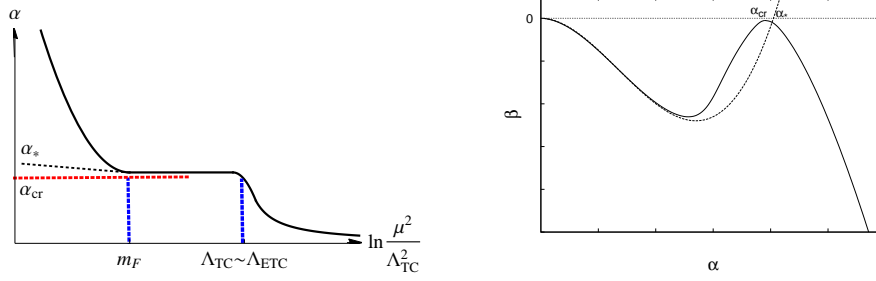


Fig. 1 Schematic picture of the running coupling (left) and the beta function (right) in the region, with the (quasi-)infrared fixed point α_* and the critical coupling as a (quasi-)ultraviolet fixed point α_{cr} . Perturbative coupling in $\alpha < \alpha_{cr}$ is smoothly connected to the nonperturbative one in the region $\alpha > \alpha_{cr}$.

4. Hidden Symmetries in the SM Higgs Lagrangian [17]

Here we recapitulate Ref. [17] to show that the SM Higgs Lagrangian Eq.(1) in the form of the *linear sigma model*, Eqs.(2) and (3), is rewritten into precisely the form equivalent to the *scale-invariant* version of the chiral $SU(2)_L \times SU(2)_R$ *nonlinear sigma model* based on the manifold G/H , with $G = SU(2)_L \times SU(2)_R$ and $H = SU(2)_{L+R} = SU(2)_V$, as far as it is in the broken phase, with *both the chiral and scale symmetries spontaneously broken* due to the same Higgs VEV $v \neq 0$, and thus are *both nonlinearly realized*.

The SM Higgs Lagrangian is further shown to be *gauge equivalent* to the *scale-invariant* version [21] of the Hidden Local Symmetry (HLS) Lagrangian [18–20], which contains *possible new vector bosons*, analogues of the ρ mesons, as the gauge bosons of the (spontaneously broken) HLS *hidden behind the SM Higgs Lagrangian*.

Let us discuss the Higgs Lagrangian in the form of Eqs.(2) and (3): The potential minimum exists at the chiral-invariant circle:

$$\langle \sigma^2(x) \rangle = \frac{-\mu_0^2}{\lambda} \equiv v^2, \quad \sigma^2(x) \equiv \hat{\sigma}^2(x) + \hat{\pi}_a^2(x). \quad (38)$$

In Eq.(3) any complex matrix M can be decomposed into the Hermitian (always diagonalizable) matrix H and unitary matrix U as $M = HU$ (“polar decomposition”):

$$M(x) = H(x) \cdot U(x), \quad H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma(x) & 0 \\ 0 & \sigma(x) \end{pmatrix}, \quad U(x) = \exp \left(\frac{2i\pi(x)}{F_\pi} \right), \quad (39)$$

with $\pi(x) = \pi^a(x) \frac{\tau^a}{2}$ ($a = 1, 2, 3$) and $F_\pi = v = \langle \sigma(x) \rangle$. The chiral transformation of M is inherited by U ,⁶ while H is a chiral singlet such that:

$$U \rightarrow g_L U g_R^\dagger, \quad H \rightarrow H, \quad (40)$$

where $g_{L/R} \in SU(2)_{L/R}$ and $U U^\dagger = 1$ implies $\langle U \rangle = \langle \exp \left(\frac{2i\pi(x)}{F_\pi} \right) \rangle = 1 \neq 0$, namely the spontaneous breaking of the chiral symmetry is taken granted in the polar decomposition. Note that *the radial mode σ is a chiral-singlet in contrast to $\hat{\sigma}$ which is a chiral non-singlet* transformed into the chiral partner $\hat{\pi}_a$ by the chiral rotation. The *physical particles are σ and π* which are defined by the nonlinear realization, *in contrast to the tachyons $\hat{\sigma}$ and $\hat{\pi}_a$* .

We further parametrize $\sigma(x)$ as

$$\sigma(x) = v \cdot \chi(x), \quad \chi(x) = \exp \left(\frac{\phi(x)}{F_\phi} \right), \quad (41)$$

where $F_\phi = v$ is the decay constant of the dilaton ϕ as the Higgs. The scale (dilatation) transformations for these fields are

$$\delta_D \sigma = (1 + x^\mu \partial_\mu) \sigma, \quad \delta_D \chi = (1 + x^\mu \partial_\mu) \chi, \quad \delta_D \phi = F_\phi + x^\mu \partial_\mu \phi. \quad (42)$$

Note that $\langle \sigma(x) \rangle = v \langle \chi(x) \rangle = v \neq 0$ breaks spontaneously the scale symmetry, but not the chiral symmetry, since $\sigma(x)$ ($\chi(x)$ as well) is a chiral singlet. This is a nonlinear realization of the scale symmetry: the $\phi(x)$ is a dilaton, NG boson of the spontaneously broken scale symmetry. Although χ is a dimensionless field, it transforms as that of dimension 1, while ϕ having dimension 1 transforms as the dimension 0, instead.

Plugging Eqs.(39) and (41) into the SM Higgs Lagrangian Eq.(3), we straightforwardly arrive at the SM Higgs Lagrangian in the striking form:[17]

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &= \left[\frac{F_\phi^2}{2} (\partial_\mu \chi)^2 + \frac{F_\pi^2}{4} \chi^2 \cdot \text{tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) \right] - V(\phi) \\ &= \chi^2(x) \cdot \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{F_\pi^2}{4} \text{tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) \right] - V(\phi), \\ V(\phi) &= \frac{\lambda}{4} v^4 \left[(\chi^2(x) - 1)^2 - 1 \right] = \frac{M_\phi^2 F_\phi^2}{8} \left[(\chi^2(x) - 1)^2 - 1 \right], \end{aligned} \quad (43)$$

which is nothing but *the scale-invariant nonlinear sigma model* with $F_\phi = F_\pi = v$, an effective theory of the walking technicolor [14, 16], apart from the form of the explicit scale-symmetry breaking potential $V(\phi)$ (see Eq.(82)).

The explicit scale-symmetry breaking comes only from the potential $V(\phi)$ such that $\delta_D V(\phi) = \lambda v^4 \chi^2 = -\theta_\mu^\mu$ whose scale dimension $d_\theta = 2$ (originally the tachyon mass term) instead of 4 of the walking technicolor: namely, the scale symmetry is broken only by the

⁶ The nonlinear realization was first introduced by K. Nishijima (then at Osaka City University) [40] (in $G/H = U(1)_L \times U(1)_R / U(1)_V$ case) to make the nucleon massive in a chiral invariant way using the NG boson π as $M_N \bar{\Psi}_L \Psi_R$, where the physical (massive) nucleon $\Psi_{L/R} = (\xi^\dagger \psi_L, \xi \psi_R)$ transforms as $\Psi_{L/R} \rightarrow h(\pi(x), g_{L/R}) \cdot \Psi_{L/R}$, ($h \in H$), while the original nucleon field $\psi_{L/R}$ does as $\psi_{L/R} \rightarrow g_{L/R} \cdot \psi_{L/R}$. Here the nonlinear base (ξ^\dagger, ξ) is defined by $U(x) = \xi^2(x)$ with the transformation, $(\xi^\dagger, \xi) = (e^{-i\pi/F_\pi}, e^{i\pi/F_\pi}) \rightarrow h(\pi(x), g_{L/R}) \cdot (\xi^\dagger, \xi) \cdot g_{L/R}^\dagger$, $h^\dagger h = 1$, in accord with Eq.(40). See, e.g., Ref.[19].

dimension 2 operator.⁷ This yields the mass of the (pseudo-)dilaton as the Higgs $M_\phi^2 = 2\lambda v^2$, which is in accord with the Partially Conserved Dilatation Current (PCDC) for $\partial^\mu D_\mu = \theta_\mu^\mu$:

$$M_\phi^2 F_\phi^2 = -\langle 0 | \partial^\mu D_\mu | \phi \rangle F_\phi = -d_\theta \langle \theta_\mu^\mu \rangle = 2\lambda v^4 \langle \chi^2(x) \rangle = 2\lambda v^4, \quad (44)$$

with $F_\phi = v$, where D_μ is the dilatation current: $\langle 0 | D_\mu(x) | \phi \rangle = -iq_\mu F_\phi e^{-iqx}$, or equivalently $\langle 0 | \theta_{\mu\nu} | \phi(q) \rangle = F_\phi(q_\mu q_\nu - q^2 g_{\mu\nu}/3)$.

Hence the SM Higgs as it stands is a (pseudo) dilaton, with the *mass arising from the dimension 2 operator* in the potential, which vanishes for $\lambda \rightarrow 0$:

$$M_\phi^2 = 2\lambda v^2 \rightarrow 0 \quad \left(\lambda \rightarrow 0, \ v = \sqrt{\frac{-\mu_0^2}{\lambda}} = \text{fixed} \neq 0 \right) \quad (45)$$

(“conformal limit”[17]).⁸ In fact the Higgs mass 125 GeV implies that the SM Higgs is in near conformal limit with $v = \text{fixed}$:

$$\lambda = \frac{1}{2} \left(\frac{M_\phi}{v} \right)^2 \simeq \frac{1}{2} \left(\frac{125 \text{ GeV}}{246 \text{ GeV}} \right)^2 \simeq \frac{1}{8} \ll 1. \quad (46)$$

It should be noted that $\lambda \ll 1$ (*with $v = \text{fixed} \neq 0$*) can be realized even when the underlying theory is strong coupling, particularly when the *scale symmetry is operative*, as we discuss later in both NJL type theory, Eq.(62), and the strong coupling gauge theory (walking technicolor) in the anti-Veneziano limit, Eq.(84).

On the other hand, if we take the limit $\lambda \rightarrow \infty$, then the SM Higgs Lagrangian goes over to the usual nonlinear sigma model *without scale symmetry*:

$$\mathcal{L}_{\text{NL}\sigma} = \frac{F_\pi^2}{4} \text{tr} \left(\partial^\mu U \partial_\mu U^\dagger \right), \quad \left(\lambda \rightarrow \infty, \ v = \sqrt{\frac{-\mu_0^2}{\lambda}} = \text{fixed} \neq 0 \right) \quad (47)$$

where the potential is decoupled with $\chi(x)$ frozen to the minimal point $\chi(x) \equiv 1$ ($\phi(x) \equiv \langle \phi(x) \rangle = v \neq 0$), so that the scale symmetry breaking is transferred from the potential to the kinetic term, which is no longer transformed as the dimension 4 operator. This is known to be a good effective theory (chiral perturbation theory) of the ordinary QCD which in fact lacks the scale symmetry at all, perfectly consistent with the nonlinear sigma model, Eq.(47). However, it cannot be true for the walking technicolor which does have the scale symmetry, and the effective theory must respect the symmetry of the underlying theory, in a form of the scale-invariant nonlinear sigma model Eq.(43) in the conformal limit $\lambda \rightarrow 0$.

Once rewritten in the form of Eq.(43), it is easy to see [17] that the *SM Higgs Lagrangian is gauge equivalent to the “scale-invariant HLS model” (s-HLS)*[21], a scale-invariant version

⁷ Note that *mass term of all the SM particles except the Higgs is scale-invariant*. By the electro-weak gauging as usual; $\partial_\mu U \Rightarrow \mathcal{D}_\mu U = \partial_\mu U - ig_2 W_\mu U + ig_1 U B_\mu$ in Eq.(43), we see that the mass term of W/Z is scale-invariant thanks to the dilaton factor χ , and so is the mass term of the SM fermions f : $g_Y \bar{f} h f = (g_Y v / \sqrt{2}) (\chi \bar{f} f)$, all with the scale dimension 4.

⁸ With vanishing potential, $V(\phi) \rightarrow 0$, this limit still gives an *interacting theory where the physical particles π and ϕ have derivative coupling* in the same sense as in the nonlinear chiral Lagrangian Eq.(47). It should be contrasted to the triviality limit, $\lambda \rightarrow 0$ *without fixing $v = \sqrt{\frac{-\mu_0^2}{\lambda}} \neq 0$* , which yields only a free theory of tachyons $\hat{\pi}$ and $\hat{\sigma}$. This limit should also be distinguished from the popular limit $\mu_0^2 \rightarrow 0$ with $\lambda = \text{fixed} \neq 0$, where the Coleman-Weinberg potential as the explicit scale symmetry breaking is generated by the trace anomaly (dimension 4 operator) due to the quantum loop.

of the HLS model [18–20]⁹, which *contains massive spin-1 states*, spontaneously broken HLS gauge bosons, as possible yet other composite states in some underlying theory hidden behind the SM Higgs.

The HLS can be made explicit by dividing $U(x)$ into two parts:

$$U(x) = \xi_L^\dagger(x) \cdot \xi_R(x), \quad (48)$$

where $\xi_{R,L}(x)$ transform under $G_{\text{global}} \times H_{\text{local}}$ as

$$\xi_{R,L}(x) \rightarrow h(x) \cdot \xi_{R,L}(x) \cdot g'_{R,L}{}^\dagger, \quad U(x) \rightarrow \hat{g}_L U(x) g'_{R,L}{}^\dagger \quad (h(x) \in H_{\text{local}}, g'_{R,L} \in G_{\text{global}}) \quad (49)$$

The H_{local} is a gauge symmetry of group H arising from the redundancy (gauge symmetry) how to divide U into two parts. Then we can introduce the HLS gauge boson $V_\mu(x)$ by covariant derivative as

$$D_\mu \xi_{R,L}(x) = \partial_\mu \xi_{R,L}(x) - i V_\mu(x) \xi_{R,L}(x), \quad (50)$$

which transform in the same way as $\xi_{R,L}$. Then we have covariant objects transforming homogeneously under H_{local} :

$$\begin{aligned} \hat{\alpha}_{\mu,R,L}(x) &\equiv \frac{1}{i} D_\mu \xi_{R,L}(x) \cdot \xi_{R,L}^\dagger(x) = \frac{1}{i} \partial_\mu \xi_{R,L}(x) \cdot \xi_{R,L}^\dagger(x) - V_\mu(x), \\ \hat{\alpha}_{\mu,||,\perp}(x) &\equiv \frac{1}{2} (\hat{\alpha}_{\mu,R}(x) \pm \hat{\alpha}_{\mu,L}(x)) \\ &= \begin{cases} \frac{1}{2i} \left(\partial_\mu \xi_R(x) \cdot \xi_R^\dagger(x) + \partial_\mu \xi_L(x) \cdot \xi_L^\dagger(x) \right) - V_\mu(x) = \alpha_{\mu,||}(x) - V_\mu(x) \\ \frac{1}{2i} \left(\partial_\mu \xi_R(x) \cdot \xi_R^\dagger(x) - \partial_\mu \xi_L(x) \cdot \xi_L^\dagger(x) \right) = \alpha_{\mu,\perp}(x) \end{cases}, \\ \{\hat{\alpha}_{R,L}(x), \hat{\alpha}_{||,\perp}(x)\} &\rightarrow h(x) \cdot \{\hat{\alpha}_{R,L}(x), \hat{\alpha}_{||,\perp}(x)\} \cdot h^\dagger(x). \end{aligned} \quad (51)$$

We thus have two independent invariants under the larger symmetry $G_{\text{global}} \times H_{\text{local}}$:

$$\mathcal{L}_A = v^2 \cdot \text{tr} \hat{\alpha}_\perp^2(x), \quad \mathcal{L}_V = v^2 \cdot \text{tr} \hat{\alpha}_||^2(x) = v^2 \cdot \text{tr} (V_\mu(x) - \alpha_{\mu,||}(x))^2. \quad (52)$$

Hence the scale-invariant version of the Higgs Lagrangian, Eq.(43), in the conformal limit $\lambda \rightarrow 0, v = \text{fixed}$, can be extended to the scale-invariant version having the HLS (s-HLS):[21]

$$\mathcal{L}_{\text{s-HLS}} = \chi^2(x) \cdot \left(\frac{1}{2} (\partial_\mu \phi)^2 + \mathcal{L}_A + a \mathcal{L}_V \right), \quad (53)$$

with a being an arbitrary parameter.

We now fix the gauge of HLS as $\xi_L^\dagger = \xi_R = \xi = e^{i\pi/v}$ such that $U = \xi^2$. Then H_{local} and $H_{\text{global}} (\subset G_{\text{global}})$ get simultaneously broken spontaneously (Higgs mechanism), leaving the diagonal subgroup $H = H_{\text{local}} + H_{\text{global}}$, which is nothing but the subgroup H of the original G of G/H : $H \subset G$. According to the Higgs mechanism, the HLS gauge boson $V_\mu(x)$ acquires the mass $\frac{1}{2} a (g_H v)^2 (V_\mu^a(x))^2$ through the invariant \mathcal{L}_V after rescaling the kinetic term of V_μ by the HLS gauge coupling g_H as $V_\mu(x) \rightarrow g_H V_\mu(x)$. *Obviously the vector boson mass terms are scale-invariant thanks to the nonlinear realization of the scale symmetry!* For the low energy $p^2 < M_V^2$ where the kinetic term can be ignored, the HLS gauge boson V_μ becomes

⁹ The s-HLS model was also discussed in a different context, ordinary QCD in medium.[41]

just an auxiliary field to be solved away to yield $\mathcal{L}_V = 0$. Noting that $\mathcal{L}_A = v^2 \cdot \text{tr} \hat{\alpha}_{\mu,\perp}^2(x) = v^2 \cdot \text{tr} \alpha_{\mu,\perp}^2(x) = \frac{v^2}{4} \cdot \text{tr} (\partial_\mu U \partial^\mu U^\dagger)$ by a straightforward algebraic calculation, we see that $\mathcal{L}_{\text{s-HLS}}$ in Eq. (53) is simply reduced back to the original SM Higgs Lagrangian $\mathcal{L}_{\text{Higgs}}$ in nonlinear realization, Eq.(43). Note that *the HLS gauge boson acquires the scale-invariant mass term thanks to the dilaton factor χ^2* , the nonlinear realization of the scale symmetry, in sharp contrast to the *Higgs (dilaton) which acquires mass only from the explicit breaking of the scale symmetry*.

The electroweak gauge bosons ($\in \mathcal{R}_\mu(\mathcal{L}_\mu)$) are introduced by extending the covariant derivative of Eq.(50) this time by gauging G_{global} , which is *independent of H_{local}* in the HLS extension:

$$D_\mu \xi_{R,L}(x) \Rightarrow \hat{D}_\mu \xi_{R,L}(x) \equiv \partial_\mu \xi_{R,L}(x) - iV_\mu(x) \xi_{R,L}(x) + i\xi_{R,L}(x) \mathcal{R}_\mu(\mathcal{L}_\mu). \quad (54)$$

As usual in the Higgs mechanism, the gauge bosons of gauged- H_{global} (\subset gauged- G_{global}) get mixed with the gauge bosons of HLS, leaving only the gauge bosons of the unbroken diagonal subgroup (gauged- H) = H_{local} + (gauged- H_{global}) be massless after mass diagonalization. We then finally have a gauged s-HLS version of the Higgs Lagrangian (gauged-s-HLS):

$$\mathcal{L}_{\text{s-HLS}}^{\text{gauged}} = \chi^2(x) \cdot \left[\frac{1}{2} (\partial_\mu \phi)^2 + \hat{\mathcal{L}}_A + a \hat{\mathcal{L}}_V \right], \quad (55)$$

with

$$\hat{\mathcal{L}}_{A,V} = \mathcal{L}_{A,V} \left(D_\mu \xi_{R,L}(x) \Rightarrow \hat{D}_\mu \xi_{R,L}(x) \right). \quad (56)$$

The new HLS boson may be identified with the LHC diboson events [23], with the model parameter choice consistent with the reported results [22], similarly to the walking technirho which is also the HLS boson for the large chiral symmetry ($SU(8)_L \times SU(8)_R$ in the one-family model) [17, 29].

A salient feature of the new vector boson of HLS in the scale-invariant SM Lagrangian is that the scale-invariant vector boson mass terms in Eq. (55) having the ϕ (Higgs H) field in the overall conformal factor $\chi^2(x)$ yield ϕ couplings only to the diagonal pairs of the (longitudinal) SM gauge bosons $H - W/Z - W/Z$ or of those of the new vector bosons $H - V - V$ after the mass diagonalization (as it should be done). Thus the new HLS vector bosons hidden in the SM only couple to the Higgs in a pair of themselves as $V - V - H$ but not in the off-diagonal combination with the SM weak bosons W/Z :

$$V - W/Z - H \text{ coupling} = 0, \quad (57)$$

namely, the decay $V \rightarrow W/Z + H$ is forbidden by the scale/conformal symmetry (*Conformal Barrier*) [17], in sharp contrast to the popular “equivalence theorem” which implies comparable coupling of V to $W/Z + W/Z$ and $W/Z + H$, based on the usual (non scale-invariant) Higgs field identification $\hat{\sigma} = v + H$, with the Higgs H being on the same footing as the NG modes $\hat{\pi}$ which are the longitudinal modes of W/Z by the equivalence theorem. Consequently, the V predominantly decays to the weak boson pairs WW/WZ . In other words, the popular consequence of the “equivalence theorem” is invalidated by the scale/conformal symmetry. The absence of $V \rightarrow WH/ZH$ signatures at the LHC Run-II thus could indirectly probe the existence of the (approximate) scale/conformal invariance of the system involving V, W, Z and H .

It is straightforward to extend the internal symmetry group to $G_{\text{global}} = SU(N_F)_L \times SU(N_F)_R$ and $H_{\text{local}} = SU(N_F)_V$. The Lagrangian then takes the form

$$\mathcal{L}_{\text{s-HLS}} = \chi^2(x) \cdot \left(\frac{1}{2} (\partial_\mu \phi)^2 + F_\pi^2 \left[\text{tr}[\hat{\alpha}_{\mu\perp}^2] + a \text{tr}[\hat{\alpha}_{\mu||}^2] \right] \right) + \dots, \quad (58)$$

where F_π is related to $v = 246$ GeV as $F_\pi = v/\sqrt{N_F/2}$. This form of the Lagrangian is the same as that of the effective theory of the one-family ($N_F = 8$) walking technicolor [21], except for the shape of the scale-violating potential $V(\phi)$ which has a scale dimension 4 (trace anomaly) in the case of the walking technicolor instead of 2 of the SM Higgs case (Lagrangian mass term). We shall come back to this later.

5. Strong Coupling NJL as the UV completion of the Weak Coupling SM Higgs Lagrangian [24]

Let us now recapitulate Ref. [24] which elaborated the composite Higgs model based on the strong coupling theory $G > G_{\text{cr}} \neq 0$ pioneered by Nambu. In the NJL model [2] for the N_C -component 2-flavored fermion ψ the Lagrangian takes the form:

$$\begin{aligned} \mathcal{L}_{\text{NJL}} &= \bar{\psi} i \gamma^\mu \partial_\mu \psi + \frac{G}{2} [(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau^a \psi)^2] \\ &= \bar{\psi} (i \gamma^\mu \partial_\mu + \hat{\sigma} + i \gamma_5 \tau^a \hat{\pi}_a) \psi - \frac{1}{2G} (\hat{\sigma}^2 + \hat{\pi}_a^2), \end{aligned} \quad (59)$$

where the equations of motion of the auxiliary fields $\hat{\sigma} \sim G \bar{\psi}\psi$ and $\hat{\pi}^a \sim G \bar{\psi} i \gamma_5 \tau^a \psi$ are plugged back into the Lagrangian to get the original Lagrangian. In the large N_C limit ($N_C \rightarrow \infty$ with $N_C G \neq 0$ fixed), after rescaling the induced kinetic term to the canonical one, $Z_\phi^{1/2} \hat{\sigma} \rightarrow \hat{\sigma}$, the quantum theory for $\hat{\sigma}$ and $\hat{\pi}$ sector yields precisely the same form as the SM Higgs Eq.(1), with: [42]

$$\begin{aligned} \mu_0^2 &= \left(\frac{1}{G} - \frac{1}{G_{\text{cr}}} \right) Z_\phi^{-1} = -2m_F^2 = -v^2 Z_\phi^{-1} = -\lambda v^2 < 0 \quad (G > G_{\text{cr}} = \frac{4\pi^2}{N_C \Lambda^2}) \\ \lambda &= Z_\phi Z_\phi^{-2} = Z_\phi^{-1} = \left[\frac{N_C}{8\pi^2} \ln \frac{\Lambda^2}{m_F^2} \right]^{-1} \sim \left[\frac{N_C}{8\pi^2} \ln \frac{\Lambda^2}{v^2} \right]^{-1}, \end{aligned} \quad (60)$$

where the gap equation has been used:

$$\frac{1}{G} - \frac{1}{G_{\text{cr}}} = -\frac{N_C}{4\pi^2} m_F^2 \ln \frac{\Lambda^2}{m_F^2} = -2m_F^2 Z_\phi = -F_\pi^2 = -v^2. \quad (61)$$

Eq.(60) shows that the *tachyon with $\mu_0^2 < 0$ is in fact generated* by the dynamical effects for the *strong coupling* $G > G_{\text{cr}} \neq 0$, corresponding to the generation of mass $m_F \neq 0$ in the gap equation. Or, we can explicitly see it by computing the $\bar{\psi}\psi$ bound state using the $m_F = 0$ solution (wrong solution) of the gap equation at $G > G_{\text{cr}}$. The correct spectrum $M_\pi^2 = 0, M_\phi^2 = 2\lambda v^2 = -2\mu_0^2 = 4m_F^2$ can be obtained when we use the correct solution $m_F \neq 0$ in the gap equation. The last equality $M_\phi^2 = 4m_F^2$, often dubbed “BCS mass relation”, is specific to the $N_C \rightarrow \infty$ (with $N_C G \neq 0$ fixed) limit of the NJL model (“*weak coupling*” limit $G > G_{\text{cr}} \sim 1/N_C \rightarrow 0$ in the *strong coupling phase*), but not the general outcome of the NJL model nor the generic linear sigma model.

There are two extreme limits for λ in Eq.(60): $\lambda \rightarrow 0$ ($N_C \gg 1$ and/or $\Lambda/v^2 \gg 1$) reproduces precisely the conformal limit, or scale-invariant nonlinear sigma model limit, Eq.(43),

of the SM Higgs Lagrangian, while $\lambda \rightarrow \infty$ ($N_C, \Lambda^2/v^2 = \mathcal{O}(1)$) does the nonlinear sigma model limit without scale symmetry, Eq.(47).

We are particularly interested in the limit

$$\lambda = \frac{1}{\frac{N_C}{8\pi^2} \ln \frac{\Lambda^2}{v^2}} \rightarrow 0 \quad (62)$$

(conformal limit in Eq.(45)), which is realized in the strong coupling theory with $G > G_{\text{cr}} \neq 0$ for $\Lambda/v \rightarrow \infty$ and/or $N_C \rightarrow \infty$, with $v = F_\pi = F_\phi \neq 0$ fixed.¹⁰ Then the effective Lagrangian in the large N_C limit takes precisely the same as the SM Higgs Lagrangian, which is further equivalent to the scale-invariant nonlinear sigma model, Eq.(43), as mentioned before. Now the SM Higgs is identified with the composite (pseudo-)dilaton with mass vanishing $M_\phi^2 = 2\lambda v^2 \rightarrow 0$.

The limit theory gives an *interacting (nontrivial) low energy effective theory even in the $\Lambda/v \rightarrow \infty$ limit*: a scale-invariant nonlinear sigma model [14, 16] where massless π and ϕ are *interacting with each other* with the (derivative) couplings $\sim (1/F_\pi, 1/F_\phi) \neq 0$. It is actually the basis for the *scale-invariant chiral perturbation theory* (sChPT) with the derivative expansion as a loop expansion [45], although the Yukawa couplings of π, ϕ to the fermions are vanishing $g_Y \sim m_F/F_\pi, m_F/F_\phi \rightarrow 0$ (The composite particles are still interacting due to the loop divergence compensation of the vanishing Yukawa coupling). This limit should be sharply distinguished from a similar limit $\Lambda/m_F \rightarrow \infty$, $m_F = \text{fixed}$ (not $\Lambda/v \rightarrow \infty$, $v = \text{fixed}$), which is the famous triviality limit (Gaussian fixed point) where the theory becomes a free theory: free massive scalar for $G < G_{\text{cr}}$ and free tachyon for $G > G_{\text{cr}}$, with not just the Yukawa couplings but all the couplings vanishing.

One might wonder why dilaton in NJL model? Obviously the NJL model has the explicit scale-breaking coupling G having dimension $[M]^{-2}$. But this scale is an ultraviolet scale to which the low energy effective theory is insensitive. This is in exactly the same sense as in the scale-invariant ladder gauge theory, Eqs.(27) and (29), where the intrinsic scale Λ_{TC} generated by the trace anomaly can be far bigger than the infrared scale of spontaneous symmetry breaking $F_\pi, F_\phi = \mathcal{O}(v) \ll \Lambda_{\text{TC}}$ thanks to the approximate scale symmetry due to the almost nonrunning coupling.

In fact we can formulate the nonperturbative running of the (dimensionless) four-fermion coupling $g = \frac{\Lambda^2}{4\pi^2} G$ in the same way as the Miransky nonperturbative renormalization: The gap equation Eq. (61) reads

$$\left(\frac{1}{g_{\text{cr}}} - \frac{1}{g} \right) \Lambda^2 = N_C m_F^2 \ln \frac{\Lambda^2}{m_F^2} \simeq 4\pi^2 v^2, \quad g_{\text{cr}} = \frac{1}{N_C}, \quad (63)$$

which leads to a nonperturbative beta function for $g > g_{\text{cr}}$:

$$\beta(g) = \Lambda \frac{\partial g(\Lambda)}{\partial \Lambda} \bigg|_{v=\text{fixed}} = -\frac{2}{g_{\text{cr}}} g \cdot (g - g_{\text{cr}}), \quad g(\mu) = g_{\text{cr}} \frac{1}{1 - \frac{4\pi^2 g_{\text{cr}} v^2}{\mu^2}} \quad (64)$$

¹⁰ If Λ is regarded as a physical cutoff in contrast to the nonperturbative renormalization arguments below, this argument would not be realistic for the 125 GeV Higgs with $\lambda \simeq 1/8$, corresponding to $\Lambda \simeq v \cdot e^{32\pi^2/N_C} \gg 10^{19}$ GeV. For the NJL model with N_D doublets, however, we would have $\Lambda \simeq v \cdot e^{32\pi^2/(N_D N_C)} \sim 10^{11}$ GeV for $N_D = N_C = 4$, somewhat realistic if the condensate is mainly triggered by the strong (ETC-induced) four-fermion coupling rather than the technicolor gauge coupling [11, 43] in the one-family technicolor model [44] ($N_F = 2N_D = 8$, see later discussions.).

by *fixing* $v = \text{constant}$ (instead of the conventional limit with $m_F = \text{constant}$) and taking $\Lambda \rightarrow \infty$. Thus, without troublesome log factor, $g = g_{\text{cr}} = 1/N_C$ is the ultraviolet fixed point defining a nontrivial interacting theory in the continuum limit. As the running coupling $g(\mu)$ reaches g_{cr} even much faster than the walking coupling in Eq.(31), the scale symmetry is operative $g(\mu) \approx g_{\text{cr}}$ for the wide region $4\pi^2 g_{\text{cr}} v^2 \ll \mu^2 < \Lambda^2$.

We also have $-\langle \bar{\psi}_i \psi_j \rangle = \delta_{i,j} \Lambda^2 m_F N_C / (4\pi^2) = Z_m^{-1} \delta_{i,j} v^3 N_C / (4\pi^2)$, where $Z_m^{-1} = Z_m^{-1}(\Lambda/v) = (\Lambda/v)^2 [N_C \ln(\Lambda^2/v^2) / (4\pi^2)]^{-1/2}$ is the mass renormalization constant, which implies¹¹

$$\gamma_m = Z_m \Lambda \frac{\partial Z_m^{-1}}{\partial \Lambda} = 2 - 1/\ln(\Lambda^2/v^2) \longrightarrow 2 \quad (\Lambda/v \rightarrow \infty). \quad (65)$$

Thus we may write $\bar{\psi}_i \psi_j = -Z_m^{-1} \delta_{i,j} v^3 N_C / (4\pi^2) \cdot \chi$, or $(G/2)(\bar{\psi}\psi)^2 = 2g N_C \Lambda^2 v^2 \cdot \chi^2 / [\ln(\Lambda^2/v^2)]$. The gap equation implies $\beta(g)/g = -g(4\pi^2)(v^2/\Lambda^2)$. Putting all together, we have $\beta(g)/g \cdot (G/2) \cdot (\bar{\psi}\psi)^2|_{g \rightarrow g_{\text{cr}}=1} = -\lambda v^4 \chi^2$. Then we get the *explicit scale symmetry breaking in the dimension 2 operator*: $\theta_\mu^\mu = \frac{\beta(g)}{g} \frac{G}{2} \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau^a \psi)^2 \right] = -\lambda v^4 \chi^2$, where $\lambda = 8\pi^2 / [N_C \ln(\Lambda^2/v^2)] \rightarrow 0$ as in Eq.(60).

The PCDC follows in precisely the same way as in the SM Higgs as $M_\phi^2 F_\phi^2 = -d_{(\bar{\psi}\psi)^2} \langle \theta_\mu^\mu \rangle = 2\lambda v^4$ (See below Eq.(43)). In any case the trace of energy-momentum tensor vanishes in the limit $\lambda \sim 1/[N_C \ln(\Lambda^2/v^2)] \rightarrow 0$, and the dilaton mass should come from the trace anomaly in the $1/N_C$ sub-leading loop effects, or the chiral loops of the effective theory Eq.(43).

Again the spin 1 composites can also be introduced via HLS, precisely in the same way as Eq.(53) for the SM Higgs Lagrangian. This time it can be done more explicitly by introducing the vector/axialvector type four-fermion coupling which in fact become the “explicit” composite HLS gauge bosons. (See section 5.3 of Ref.[19]).

Incidentally, the above prescription to have an interacting nontrivial continuum theory of the NJL model is similar to the renormalizability arguments of the D -dimensional NJL model ($2 < D < 4$) [31] and the gauged NJL model [25] both without troublesome log factor, although in the case at hand the explicit scale-breaking from the Lagrangian parameters, i.e., the four-fermion interaction and fermion mass term (if present), depend on the renormalization point (vanish at the UV limit).

The beta function Eq.(64) and anomalous dimension Eq.(65) of the D -dimensional four-fermion theory renormalizable in $1/N_C$ expansion are given: [31]

$$\beta(g) = -\frac{D-2}{g_{\text{cr}}} g (g - g_{\text{cr}}), \quad \gamma_m = (D-2) \frac{g}{g_{\text{cr}}} \rightarrow D-2 \quad (g \rightarrow g_{\text{cr}}), \quad (66)$$

which follows from the gap equation Eq.(16) and the condensate $\langle \bar{\psi}\psi \rangle \sim (\Lambda/m_F)^{D-2}$, respectively, while the renormalization can be done independently of the phase in this theory.

¹¹ Hence the operators have scale dimension $d_{\bar{\psi}\psi} = 1$ and $d_{(\bar{\psi}\psi)^2} = 2$ in the large N_C limit. Note that γ_m is actually slightly smaller by the $1/\ln(\Lambda^2/v^2)$ than “ $\gamma_m = 2$ ” in the conventional limit $m_F = \text{constant}$, so that $d_{(\bar{\psi}\psi)^2}$ is slightly larger than 2 actually, i.e., possible eight-fermion operators corresponding to the $\lambda\phi^4$ would have dimension $d_{(\bar{\psi}\psi)^4} > 4$, barely irrelevant. This is contrasted to the conventional limit where the eight-fermion operators are marginal and hence the NJL coupling without such counter terms would not be renormalizable nor interacting theory in the continuum limit (See the section 8 of the 3rd entry of Ref.[25]).

For $D = 2$ the ultraviolet fixed point $g = 0$ and the infrared fixed point $g = g_{\text{cr}}$ coincide, i.e.,

$$\beta(\alpha) \rightarrow -2N_C g^2, \quad \gamma_m \rightarrow 2N_C g, \quad (67)$$

which also follows directly from Eq.(18). The essential singularity scaling in Eq.(18) and the associated colliding ultraviolet and infrared fixed points at $D = 2$ are characteristic features of the conformal phase transition [28] similarly to the BKT-Miransky scaling, where there exist no light bound states for $g < g_{\text{cr}} = 0$, while the bound states have mass of order of $\mathcal{O}(m_F) \rightarrow 0$ ($g \searrow g_{\text{cr}} = 0$) for $g > g_{\text{cr}}$.

It is also compared with the beta function of the gauged NJL model with walking gauge coupling $\alpha(\mu) = \alpha = \text{constant}$: [25]

$$\beta(g) = \left. \frac{\partial g}{\partial \ln \Lambda} \right|_{\alpha, m_F} = -2N_C \left(g - g^{(+)}(\alpha) \right) \left(g - g^{(-)}(\alpha) \right), \quad (68)$$

where [46, 47]

$$N_C g^{(\pm)}(\alpha) = \frac{1}{4} (1 \pm \omega)^2, \quad \omega \equiv \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}} \quad (0 < \alpha < \alpha_{\text{cr}}). \quad (69)$$

This follows from the gap equation (for $m_0 = 0$):

$$\Sigma(x) = \frac{N_C G}{4\pi^2} \int^{\Lambda^2} dx \frac{x \Sigma(x)}{x + \Sigma(x)} + \frac{3C_2}{4\pi} \alpha \int^{\Lambda^2} dy \left[\frac{\theta(x-y)}{x} + \frac{\theta(y-x)}{y} \right] \frac{y \Sigma(y)}{y + \Sigma^2(y)}, \quad (70)$$

which is a combined gap equation of Eq.(10) and Eq.(20). Were it not for the NJL interaction $G = 0$, the SSB solution would exist only for $\alpha > \alpha_{\text{cr}}$ as we have explained in section 3. However, due to strong NJL coupling, this time it has the SSB solution for $g = G\Lambda^2/(4\pi^2) > g^{(+)}(\alpha)$ even at $\alpha < \alpha_{\text{cr}}$: [46, 47]

$$m_F^{2\omega} = \Lambda^{2\omega} \left(\frac{g - g^{(+)}(\alpha)}{g - g^{(-)}(\alpha)} \right), \quad \left(g > g^{(+)}(\alpha) = g_{\text{cr}}(\alpha), \quad 0 < \alpha < \alpha_{\text{cr}} \right) \quad (71)$$

as well as $\alpha > \alpha_{\text{cr}}$, while $g < g^{(+)}(\alpha)$ is the unbroken phase, $m_F = 0$, where the criticality is now extended to the critical line $g_{\text{cr}}(\alpha) = g^{(+)}(\alpha)$ in the (g, α) space instead of just the gauge coupling α . The beta function Eq.(68) is readily obtained from Eq.(71), and the anomalous dimension is given as: [11, 25]

$$\gamma_m = 2N_C g + \frac{\alpha}{2\alpha_{\text{cr}}}, \quad \gamma_m^{(\pm)} = \gamma_m \Big|_{g=g^{(\pm)}(\alpha)} = 1 \pm \omega = 1 \pm \sqrt{1 - \frac{\alpha}{\alpha_{\text{cr}}}}. \quad (72)$$

The critical line $g = g_{\text{cr}}(\alpha) = g^{(+)}(\alpha)$, coming from the four-fermion coupling additional to the gauge dynamics (e.g., ETC coupling in the technicolor case), behaves as an ultraviolet fixed point, while the non-critical line $g^{(-)}(\alpha)$, coming from the four-fermion coupling induced by the gauge dynamics itself in a Wilsonian sense, does as an infrared fixed point, both colliding $\bar{g}_{\text{cr}} = g^{(+)}(\alpha) = g^{(-)}(\alpha) = 1/(4N_C)$ at $\alpha = \alpha_{\text{cr}}$, where the SD gap equation yields the Miransky-BKT scaling of the essential-singularity form (conformal phase

transition):

$$m_F = \Lambda \exp \left(-\frac{g}{g - \bar{g}_{\text{cr}}} \right), \quad \left(\alpha = \alpha_{\text{cr}}, g > \bar{g}_{\text{cr}} = \frac{1}{4N_C} \right). \quad (73)$$

Accordingly, the beta function and the anomalous dimension read

$$\beta(g) = -2N_C (g - \bar{g}_{\text{cr}})^2, \quad \gamma_m = 2N_C g + \frac{1}{2} \quad \left(\gamma_m \Big|_{\bar{g}_{\text{cr}}} = 1 \right), \quad (74)$$

which is compared with Eq.(67).

The outstanding feature of the gauged NJL model with $0 < \alpha \leq \alpha_{\text{cr}}$, ($1 \leq \gamma_m < 2$) is the renormalizability (in the sense of nontriviality, or no Landau pole) independently of the phase [25], when the gauge coupling is walking, $\alpha(\mu^2) \approx \text{const.}$ (Similar results are obtained for the gauged Yukawa model [48].) The four-fermion operators have the full dimension $2 < d_{(\bar{\psi}\psi)^2} = 2(3 - \gamma_m) = 4 - 2\omega \leq 4$ (relevant operator, or super renormalizable), including $d \simeq 2(1 + A/\ln \mu^2) > 2$ ($d \neq 2$) with a moderately “walking” small coupling $\omega \simeq 1 - \frac{\alpha}{2\alpha_{\text{cr}}} \simeq 1 - \gamma_m$ ($\gamma_m(\mu) \sim A/\ln \mu^2$) with $A = 18C_2/(11N_C - 2N_F) > 1$ ¹², in sharp contrast to the pure (non-gauged) NJL model with $\gamma_m = 2$, $d_{(\bar{\psi}\psi)^2} = 2$, which is a trivial theory having a Landau pole in the conventional way of the continuum limit keeping $m_F = \text{constant}$ (not the way described in the above keeping $F_\pi = \text{constant}$).¹³

6. Top Quark Condensate a la NJL dynamics, the simplest UV completion of the SM Higgs

One of the concrete composite Higgs models as the straightforward application of the NJL type theory is the top quark condensate model (Top-Mode Standard Model) [8–10]. The model predicted that only the top quark among SM fermions has mass on the order of the weak scale (Fig.2), at the time many people expected the top mass below 50 GeV.

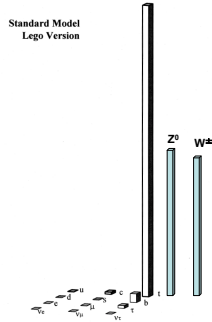


Fig. 2 Lego version of the Standard Model. Top quark mass vs. other masses in linear scale.

¹² For the NJL gauged by the actual QCD with 6 flavors (u, \dots, t), $A = 8/7 > 1$, as in the original top quark condensate model [8] satisfies this renormalizability condition [25], though the electroweak gauge interaction invalidates it. If the SM gauge groups are embedded into a GUT which is usually walking $A > 1$, then the GUT-gauged NJL is renormalizable, having the interacting continuum limit.

¹³ For renormalizability of the gauge theories and gauged NJL model in $D > 4$ dimensions, with the extra dimensions $\delta = D - 4$ compactified, see Ref.[49].

The explicit four-fermion Lagrangian of the top quark condensate takes the form: [8]

$$\begin{aligned}\mathcal{L}_{\text{MTY}} = & G^{(1)} \left(\bar{\psi}_L^i \psi_R^j \right) \left(\bar{\psi}_R^j \psi_L^i \right) + \left[G^{(2)} \left(\bar{\psi}_L^i \psi_R^j \right) (i\tau_2)^{ik} (i\tau_2)^{jl} \left(\bar{\psi}_L^k \psi_R^l \right) + h.c. \right] \\ & + G^{(3)} \left(\bar{\psi}_L^i \psi_R^j \right) (\tau_3)^{jk} \left(\bar{\psi}_R^k \psi_L^i \right), \quad \left(G^{(i)} = g^{(i)} \frac{4\pi^2}{\Lambda^2} \right). \quad (75)\end{aligned}$$

The inclusion of other generations is straightforward [8]. In the realistic case the SM gauge interaction, particularly QCD, was included via the gauged NJL model [8], where the critical coupling is actually the critical line of the gauged NJL, $G_{\text{cr}}(\alpha) = g_{\text{cr}}(\alpha) \cdot (4\pi^2/\Lambda^2)$ with $g_{\text{cr}}(\alpha)$ in Eq.(69), while the $U(1)_Y$ coupling is numerically negligible (the chiral gauge $SU(2)_L$ does not contribute to the condensate channel). The crucial ingredient of the model is again the *non-zero critical coupling in sharp contrast to the “bootstrap symmetry breaking”* [9] based on the weakly-coupled BCS theory which has $G_{\text{cr}} = 0$ as already mentioned: *only the top quark coupling is strong coupling larger than the critical coupling* $G_t = G^{(1)} + G^{(3)} > G_{\text{cr}}(\alpha)$ while $G_b = G^{(1)} - G^{(3)}$ and all others are less as well, $G_{c,s,d,u} < G_{\text{cr}}(\alpha)$, so that only the top acquires the dynamical mass of order of weak scale $\mathcal{O}(v)$ to produce only three NG bosons to be absorbed into the W/Z bosons [8, 10].

We disregard $G^{(2)}$ and G_b terms for the moment, then Eq.(75) simply reads:

$$\begin{aligned}\mathcal{L}_{\text{MTY}} = & G_t \left(\bar{\psi}_L t_R \right) \left(\bar{t}_R \psi_L \right) \\ = & \bar{\psi}_L h t_R + h.c. - \frac{1}{G_t} h^\dagger h, \quad (76)\end{aligned}$$

where $h = G_t \bar{\psi}_L t_R$ with $\hat{\sigma} \sim G_t \bar{t} t$, $\hat{\pi}^0 \sim G_t \bar{t} i \gamma_5 t$, $\hat{\pi}^\pm \sim G_t \bar{t} i \gamma_5 b$, $G_t \bar{b} i \gamma_5 t$. This simplified version was also considered in [10].

The effective theory of the pure bosonic sector at $1/N_C$ leading order is precisely the same as the SM Higgs Lagrangian Eq.(1) [10] as already mentioned in Section 5, which happens to have the $SU(2)_L \times SU(2)_R$ global symmetry not just the $SU(2)_L \times U(1)_R$ to be gauged by the electroweak gauge bosons. (The $SU(2)_R$ is explicitly broken only by the Yukawa term: $\bar{\psi}_L h t_R$.) Then the effective theory of the top quark condensate model is nothing but the SM Higgs as the scale-invariant HLS model, Eq.(53), which includes the 3 NG bosons $\pi^{\pm,0}$ to be absorbed into W, Z and the Higgs $\phi(x)$ as a pseudo-dilaton, in the nonlinear realization in Eq.(39) and (41), and in addition the vector composites (“top-mode rho meson”) which can be identified [22] with the 2 TeV diboson events at LHC [23].

If we further assume a small bottom condensate $\langle \bar{b} b \rangle$ by fine-tuning the bottom four-fermion coupling: $G_t > G_b > G_{\text{cr}}(\alpha)$, then we have a Peccei-Quinn type axion (“top-mode axion”) [8, 50], which acquires mass from the $G^{(2)}$ term in Eq.(75). (It may be identified with the 750 GeV diphoton events at LHC [26].)

The obvious phenomenological problem of the top-mode SM is the prediction of the top mass in the large N_C limit relation (BCS mass relation) to the Higgs mass: $M_\phi = 2m_t$. which is actually modified by the effects of the SM gauge interactions, $M_\phi \simeq \sqrt{2}m_t$ [10, 51]. It is further modified to $M_\phi \simeq m_t$ by some of the non-leading order in $1/N_C$ expansion [10] using the ultraviolet boundary condition, “compositeness condition” [10], at say the GUT scale, where both effective couplings of top Yukawa g_Y^t and the Higgs quartic coupling λ diverge.

Instead of the compositeness condition, we may consider the renormalizability of the gauged NJL model mentioned in the previous section. A possible renormalizable top quark condensate model would then be to unify the SM gauge interactions into a walking GUT,

“Top-mode walking GUT” [52], which determines the values of the top Yukawa coupling g_Y^t and the Higgs coupling λ in terms of the GUT gauge coupling g_{GUT} all at the GUT scale Λ_{GUT} as the Pendleton-Ross infrared fixed point of the effective theory of the GUT-gauged NJL model, typically as $g_Y^t{}^2(\Lambda_{\text{GUT}}) \simeq \lambda(\Lambda_{\text{GUT}}) \simeq \frac{3}{2} g_{\text{GUT}}^2(\Lambda_{\text{GUT}})$, instead of the diverging couplings of the compositeness condition. This generally yields prediction of mass of m_t and M_ϕ much smaller than that of compositeness condition.

Another possibility would be to include the near marginal operators: since the anomalous dimension is very close to 2, the four-fermion operators (corresponding to the bare $\lambda\phi^4$ term) have the dimension $d \simeq 2$, so that formally irrelevant eight-fermion operators could be near marginal and compete with the higher order corrections in $1/N_C$ expansion, which may change the mass ratio substantially. It is not known presently whether or not the relation $M_\phi \simeq m_t/2$ can be naturally realized by yet higher order effects in $1/N_C$ as well as the eight-fermion operators.

Yet other different solutions have been considered, see e.g. top seesaw [53], and its NG boson Higgs version [54] where the Higgs is a pseudo NG boson living in the coset space of the larger internal symmetry $G/H = U(3) \times U(1)/[U(2) \times U(1)']$ rather than the pseudo-dilaton.

The LHC Run II will tell us whether or not the basic idea of the top quark condensate is on the right track.

7. Walking Technicolor and Technidilaton

Yet another composite Higgs model in the spirit of Nambu is the strong coupling gauge theory, similar in some sense to the QCD, a simple scale-up version dubbed technicolor [55]. However the original technicolor was excluded long time ago for the problem of the Flavor-Changing Neutral Currents (FCNC) in a way to give mass to the SM fermions through four-fermion interaction from Extended Technicolor (ETC) [56] or some composite technicolor models [57]. As the strong coupling gauge theory with *more explicit role of the non-zero critical coupling*, the walking technicolor [12, 13] was proposed as a solution to the FCNC problem by a large anomalous dimension $\gamma_m = 1$, based on the SSB solution of the scale-invariant dynamics, the ladder SD equation, and at the same time predicted a light composite Higgs, dubbed technidilaton, as a pseudo NG boson of the approximate scale symmetry.¹⁴

Here I recapitulate the explanation [16] on how the technidilaton is a *naturally light and weakly coupled composite Higgs* out of *strongly coupled* underlying conformal gauge theory, the walking technicolor, in the light of the anti-Veneziano limit Eq.(36). The technidilaton particularly for the one-family technicolor, with $N_F = 8$ and $N_C = 4$ [56], as the walking technicolor is nicely fit to the current 125 GeV Higgs data at LHC [14–16].

As we discussed in section 3, ladder approximation is realized by the anti-Veneziano limit Eq.(36) in the large N_F QCD. The evaluation of the nonperturbative trace anomaly in the anti-Veneziano limit can be essentially given by the ladder result Eq.(33). It then yields the mass M_ϕ and decay constant F_ϕ of the technidilaton ϕ through PCDC [13] as in Eq.(44),

¹⁴ Similar work on the FCNC problem [58] was also done without notion of the anomalous dimension, the scale symmetry, nor the technidilaton. Solving FCNC by a large anomalous dimension was proposed earlier [59], based on a pure assumption of the existence of a gauge theory having nontrivial ultraviolet fixed point.

this time in terms of the dimension 4 operator:[16, 60]

$$M_\phi^2 F_\phi^2 = -d_\theta \langle \theta_\mu^\mu \rangle = -\frac{\beta^{(\text{NP})}(\alpha(\mu^2))}{\alpha(\mu^2)} \langle G_{\mu\nu}^2(\mu^2) \rangle \simeq N_C N_F \frac{16\xi^2}{\pi^4} m_F^4 \quad (d_\theta = 4) \quad (77)$$

$$\simeq 2.5 \left[\frac{8}{N_F} \frac{4}{N_C} \right] v^4. \quad (v = 246 \text{ GeV}) \quad (78)$$

First, the rightmost value in Eq.(77) can be obtained by two different ladder calculations: one through direct evaluation of the vacuum energy by the effective potential at the stationary point (Solution of the SD equation, $\Sigma = \Sigma_{\text{sol}}$)[61], $E = V_{\text{eff}}(\Sigma = \Sigma_{\text{sol}}) = \langle \theta_0^0 \rangle = (1/4) \langle \theta_\mu^\mu \rangle$, the other through the ladder evaluation of the trace anomaly [16, 60], i.e., the technigluon condensate $\langle G_{\mu\nu}^2 \rangle$ times the nonperturbative beta function Eq.(30), both in precise agreement with each other. The agreement is in highly nontrivial manner, being *independent of the renormalization point* μ as it should be: $\langle G_{\mu\nu}^2(\mu^2) \rangle \sim \ln^3(\mu^2/m_F^2)$, while $\beta^{(\text{NP})}(\alpha(\mu^2))/\alpha(\mu^2) \sim 1/\ln^3(\mu^2/m_F^2)$, precisely cancelled by each other.[16]

Second, Eq.(78) is obtained by use of the Pagels-Stokar formula:

$$v^2 = (246 \text{ GeV})^2 = N_D F_\pi^2 \simeq N_F N_C \frac{\xi^2}{4\pi^2} m_F^2 \simeq m_F^2 \left[\frac{N_F}{8} \frac{N_C}{4} \right], \quad (79)$$

and the result indicates *important* N_F, N_C -dependence of $M_\phi^2 F_\phi^2$ in the anti-Veneziano limit when $v = \text{fixed}$ [16]. Since the technidilaton is a flavor-singlet bound state, its decay constant by definition scales like $F_\phi^2 \propto N_F N_C m_F^2 (\propto v^2)$ (Actually $F_\phi^2 \simeq N_F N_C m_F^2$). Then $M_\phi^2/F_\phi^2, M_\phi^2/v^2 \sim 1/(N_F N_C) \rightarrow 0$ in the anti-Veneziano limit, where the technidilaton becomes NG boson although no exact massless limit exists: the situation is in the same sense as the η' meson in the original Veneziano limit $N_C \rightarrow \infty$ with $N_C \alpha = \text{fixed}$, and $N_F/N_C \ll 1$.¹⁵

Although the PCDC relation together with Pagels-Stokar formula does not give M_ϕ and F_π separately, Eq.(78) well accommodates numerically the desired result:

$$F_\phi \simeq 5 v \quad \text{for } M_\phi \simeq \frac{v}{2} \simeq 125 \text{ GeV} \quad (N_F = 8, N_C = 4), \quad (80)$$

in the one-family model, which is best fit to the current LHC data of the 125 GeV Higgs up to 30 % uncertainty due to limitation of the ladder approximation [14, 16]. Similar results are also obtained within 30% uncertainty in the holographic model for the walking technicolor.[15]

Incidentally, at the criticality $\alpha = \alpha_{\text{cr}} = \frac{\pi}{3C_2}$, the anomalous dimension $\gamma_m = 1$ implies that the induced four-fermion interaction generated by the walking technicolor coupling itself (i.e., not the ETC-like gauge interaction additional to the technicolor interaction) also becomes a marginal operator with $d = 4$. Then the phase diagram should be considered in the wider coupling space (α, g) [35], where g is the dimensionless coupling of the induced four-fermion interaction in the form of the gauged NJL model. See Eq.(73) and (74). Then we can predict M_ϕ independently of F_ϕ : the denominator of the renormalized σ propagator $D^\sigma(p)$ can be

¹⁵ There exists no exact massless limit in the conformal phase transition at $\alpha = \alpha_{\text{cr}}$ (this time $r(= N_F/N_C) = r_{\text{cr}}$), with $m_F = 0$, where no massless spectrum exists for $\alpha \leq \alpha_{\text{cr}}$ (conformal phase), in sharp contrast to the Ginzburg-Landau phase transition where the spectrum continuously passes through the phase transition point with massless particles. [28]

evaluated at $p = 0$ in the large N_C limit [62]:

$$M_\phi^2 = D^\sigma(0) = \frac{16\xi^2}{\pi^4} m_F^2 \simeq \left(\frac{m_F}{2}\right)^2 \quad (\alpha = \alpha_{\text{cr}}, g \searrow \bar{g}_{\text{cr}}), \quad (81)$$

which yields $M_\phi \simeq \frac{v}{2} \simeq 125$ GeV (!) through the Pagels-Stokar formula Eq.(79) for $N_F = 8, N_C = 4$, i.e., $v \simeq m_F$, and in turn predicts $F_\phi^2 \simeq N_F N_C m_F^2$, combined with the PCDC relation Eq.(77). The result is quite consistent with Eq.(80) for the 125 GeV Higgs.

The effective theory of the walking technicolor with N_F massless flavors takes *precisely the same scale-invariant form as the nonlinearly realized SM Higgs Lagrangian* in Eq.(43), with $U = e^{i\pi^a T^a}$ being $N_F \times N_F$ unitary matrix ($\text{tr} T^a = 0, \text{tr}(T^a T^b) = \delta^{ab}/2, a = 1, \dots, N_F^2 - 1$), except that the explicit scale breaking comes from the different potential $V^{(4)}(\phi)$ ¹⁶ responsible for the *trace anomaly of dimension 4 operator* this time: [14, 16]

$$\begin{aligned} \mathcal{L}_{\text{WTC}} &= \chi^2 \cdot \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{F_\pi^2}{4} \text{tr} \left(\mathcal{D}_\mu U \mathcal{D}^\mu U^\dagger \right) \right] - V^{(4)}(\phi) - V^{(\text{SM})}(\phi), \\ V^{(4)}(\phi) &= -\ln \chi \cdot \frac{\beta^{(\text{NP})}(\alpha)}{4\alpha} G_{\mu\nu}^2 = \frac{M_\phi^2 F_\phi^2}{4} \chi^4 \left(\ln \chi - \frac{1}{4} \right) \\ V^{(\text{SM})}(\phi) &= -\chi^{2-\gamma_m} (m_f \chi \bar{f} f) - \ln \chi \left[\frac{\beta_F(\alpha_s)}{4\alpha_s} G_{\mu\nu}^{(\text{gluon})2} + \frac{\beta_F(\alpha_e)}{4\alpha_e} F_{\mu\nu}^{(\gamma)2} \right], \\ \chi &= \exp \left(\frac{\phi}{F_\phi} \right), \end{aligned} \quad (82)$$

where $F_\phi \neq F_\pi = v/\sqrt{N_D} = v/\sqrt{N_F/2}$ in general in contrast to the SM Higgs case $F_\phi = F_\pi = v$, the electroweak gauging was done as usual $\partial_\mu U \Rightarrow \mathcal{D}_\mu U = \partial_\mu U - ig_2 W_\mu U + ig_1 U B_\mu$, and we have added $V^{(\text{SM})}(\phi)$, the scale symmetry breaking terms related to the SM particles arising from the technifermion contributions: mass term of the SM fermion f , (one loop) technifermion contributions to the trace anomaly for the gluon and photon operators, with $\beta_F(g_s) = \frac{g_s^3}{(4\pi)^2} \frac{4}{3} N_C$ and $\beta_F(e) = \frac{e^3}{(4\pi)^2} \frac{16}{9} N_C$. It is obvious that $\theta_\mu^{\mu(\text{TC})} = -\delta_D V^{(4)}(\phi) = \beta^{(\text{NP})}(\alpha)/(4\alpha) \cdot G_{\mu\nu}^2 = -(M_\phi^2 F_\phi^2/4) \chi^4$ up to total derivative, corresponding to the PCDC with $d_\theta = 4$ ($\langle \chi \rangle = 1$), Eq.(77).

The technidilaton potential $V^{(4)}(\phi)$ is expanded in ϕ/F_ϕ :

$$V^{(4)}(\phi) = -\frac{M_\phi^2 F_\phi^2}{16} + \frac{1}{2} M_\phi^2 \phi^2 + \frac{4}{3} \frac{M_\phi^2}{F_\phi} \phi^3 + 2 \frac{M_\phi^2}{F_\phi^2} \phi^4 + \dots, \quad (83)$$

which shows a remarkable fact that in the anti-Veneziano limit the technidilaton self couplings (trilinear and quartic couplings) are highly suppressed:

$$\lambda_{\phi^3} = 4M_\phi^2/(3F_\phi) \sim 1/\sqrt{N_F N_C} \rightarrow 0, \quad \lambda_{\phi^4} = 2M_\phi^2/F_\phi^2 \sim 1/(N_F N_C) \rightarrow 0, \quad (84)$$

by $M_\phi/F_\phi \sim 1/\sqrt{N_F N_C}$ and $M_\phi \sim N_F^0 N_C^0$. Numerically, we may compare the technidilaton self couplings with those of the SM Higgs with $m_h = M_\phi = 125$ GeV for $v/F_\phi \simeq 1/5$ in the

¹⁶ This potential is indeed obtained by the explicit ladder computation of the effective potential at the conformal phase transition point: $V^{(4)}(\phi) = -(4N_F N_C m_F^4/\pi^4) \chi^4 (\ln \chi - 1/4)$, in precise agreement with Eq.(82) through Eq.(77). See Eq.(65) in Ref. [28]

one-family model ($N_F = 8, N_C = 4$): [16]

$$\begin{aligned} \left. \frac{\lambda_{\phi^3}}{\lambda_{h_{\text{SM}}^3}} \right|_{M_\phi=m_h} &= \left. \frac{\frac{4M_\phi^2}{3F_\phi}}{\frac{m_h^2}{2v}} \right|_{M_\phi=m_h} \simeq \frac{8}{3} \left(\frac{v}{F_\phi} \right) \simeq 0.5, \\ \left. \frac{\lambda_{\phi^4}}{\lambda_{h_{\text{SM}}^4}} \right|_{M_\phi=m_h} &= \left. \frac{\frac{2M_\phi^2}{F_\phi^2}}{\frac{m_h^2}{8v^2}} \right|_{M_\phi=m_h} = 16 \left(\frac{v}{F_\phi} \right)^2 \simeq 0.6. \end{aligned} \quad (85)$$

This shows that the *technidilaton self couplings, although generated by the strongly coupled interactions, are even smaller than those of the SM Higgs*, a salient feature of the approximate scale symmetry in the anti-Veneziano limit !!

The coupling of the technidilaton ($M_\phi = 125$ GeV) to the SM particles can be seen by expanding $\chi = 1 + \phi/F_\phi + (1/2!)(\phi/F_\phi)^2 + \dots$ in Eq.(82):

$$\frac{g_{\phi WW/ZZ}}{g_{h_{\text{SM}} WW/ZZ}} = \frac{g_{\phi ff}}{g_{h_{\text{SM}} ff}} = \frac{v}{F_\phi}. \quad (86)$$

$$\frac{g_{\phi gg}}{g_{h_{\text{SM}} gg}} \simeq \frac{v}{F_\phi} \cdot (1 + 2N_C), \quad \frac{g_{\phi \gamma\gamma}}{g_{h_{\text{SM}} \gamma\gamma}} \simeq \frac{v}{F_\phi} \cdot \left(\frac{63-16}{47} - \frac{32}{47}N_C \right), \quad (87)$$

where besides the technifermions loop, only the top and W of the SM contributions were included at one-loop. Note the couplings in Eq.(86) with $v/F_\phi \sim 1/5$ are even weaker than the SM Higgs, which are however compensated by those in Eq.(87) for gg and $\gamma\gamma$ rather enhanced by the extra loop contributions of the technifermions other than the SM particles, particularly for large N_C , resulting in signal strength similar to the SM Higgs within the current experimental accuracy.

In fact the current LHC data for 125 GeV Higgs are fit by the technidilaton as good as by the SM Higgs, particularly for $N_F = 8, N_C = 4$, i.e., near the anti-Veneziano limit. [14] Most recent detailed analyses are given in Ref. [16]. It should be mentioned here that the one-family model will be most naturally imbedded into the ETC in the case for $N_C = 4$ [63]. More precise data at LHC Run II will discriminate among them, SM Higgs or technidilaton.

What about the technipions? In the walking technicolor with $N_D = N_F/2 > 1$, the spontaneous breaking of the chiral symmetry larger than $SU(2)_L \times SU(2)_R$ produces NG bosons (technipions) more than 3 to be absorbed into W/Z. Let us take the one-family model with $N_F = 8$, which has colored techniquarks (3 weak doublets) Q_i^a and non-colored technileptons (one weak-doublet) L_i ($a = 1, 2, 3; i = 1, 2$), the resultant chiral symmetry being $SU(8)_L \times SU(8)_R$ [44]. There are 63 technipions, 60 of which are unabsorbed technipions. All of them acquire the mass from the explicit chiral symmetry breaking due to the SM gauge and ETC gauge interactions. Due to the large anomalous dimension $\gamma_m \simeq 1$, the mass of them are all enhanced to TeV region [64], which will be discovered at LHC Run II. (After this symposium, 750 GeV diphoton events were reported at LHC [26], which can be identified with the color-singlet and iso-singlet (not flavor-singlet) technipion P^0 [30, 65]. If it is the case, $M_{P^0} = 750$ GeV, the model predicts another nearby color-singlet technipion P^i (isotriplet one), with mass $M_{P^i} = \sqrt{\frac{8}{5}} M_{P^0} \simeq 950$ GeV, a salient prediction of the one-family model independently of the dynamical details [64].)

Another signature of the walking technicolor is the prediction of higher resonances such as the spin 1 boson, the walking techni- ρ , walking techni- a_1 , etc.. The straightforward N_F

extension of Eq.(53) is also obvious: Eq.(82) is gauge equivalent to the *scale-invariant HLS Lagrangian* explicitly constructed for one-family walking technicolor with $N_F = 8$ [21]:

$$\begin{aligned}\mathcal{L}_{\text{s-HLS}} &= \mathcal{L}_{\text{WTC}} + \mathcal{L}_{\text{Kinetic}}(V_\mu) , \\ \mathcal{L}_{\text{WTC}} &= \chi^2 \cdot \left(\frac{1}{2} (\partial_\mu \phi)^2 + \mathcal{L}_A + a\mathcal{L}_V \right) - V^{(4)}(\phi) ,\end{aligned}\tag{88}$$

where the HLS gauge bosons V_μ in the mass term $a\chi^2\mathcal{L}_V = ae^{2\phi/F_\phi}\text{tr}(g_{\text{HLS}}V_\mu + \dots)^2$ are the bound states of the walking technicolor, the walking techni- ρ , with *mass term* $M_V^2 = ag_{\text{HLS}}^2 F_\pi^2$ *being scale-invariant* thanks to the overall technidilaton factor χ^2 , as mentioned before. The loop expansion is formulated as the scale-invariant HLS perturbation theory [17] in the same way as the scale-invariant chiral perturbation theory [45], a straightforward extension of the (non-scale-invariant) HLS perturbation theory [20].

In fact there has been reported an interesting 2 TeV diboson event at LHC [23]. We have shown [29] that this would be the most natural candidate for the Drell-Yan produced walking techni- ρ , the color-singlet iso-triplet ρ^i , as a gauge boson of the HLS described by the scale-invariant HLS model in Eq.(88) [21]. We further found [17] that a salient feature of this possibility is the scale symmetry which forbids the decay of the walking techni- ρ to the 125 GeV Higgs (technidilaton) plus W/Z (what we called “conformal barrier”) in the same way as the hidden vector in the SM Higgs, Eq. (57), in sharp contrast to the popular “equivalence theorem”.

The HLS is readily extendable to include techni- a_1 , etc. [18–20], with an infinite set of the HLS tower being equivalent to the deconstructed extra dimension [66] and/or the holographic models [67], and the scale-invariant version of them are also straightforward and *mass term of all the higher HLS vector bosons are scale-invariant*, an outstanding characterization in sharp contrast to other formulations for the spin 1 bosons. The conformal barrier applies not only to the techni- ρ but also to all the higher vector/axialvector resonances as the HLS gauge bosons, having scale-invariant mass term. The LHC Run II will tell us whether or not it is the case.

8. Walking Technicolor on the Lattice

Finally, I would like to make a brief review on the lattice studies on the walking technicolor particularly done by our group, the LatKMI Collaboration [68–71]. Since the dynamics is essentially nonperturbative, reliable calculations would be eventually done by the lattice simulations. In fact there has been extensive activity in the lattice simulations of the candidate theories for the walking technicolor, particularly the large N_F QCD, i.e., the $SU(N_C)$ gauge theory with N_F degenerate flavors of fundamental representation fermions, eventually extrapolated to the chiral limit [72]. Among others particular interest has been paid to the $N_f = 8$ and $N_F = 12$ for $N_C = 3$, partly because the infrared fixed point α_* in the two-loop beta function exists for $N_F \gtrsim 8$ ($N_C = 3$), and the SSB criticality condition $\alpha_* = \alpha_{\text{cr}}$ for the ladder result $\alpha_{\text{cr}} = \pi/(3C_2)$ is fulfilled for $N_F \simeq 12$ [27], so that it is expected that the walking theory might exist somewhere around $8 < N_F < 12$. Inexpensive simulations are mostly done in the staggered fermion, $N_F = 4, 8, 12, 16$, within the asymptotically free cases.

The LatKMI Collaboration started in 2010 for the lattice simulations on the possible candidate for the walking technicolor by systematic studies of the $N_F = 16, 12, 8, 4$ on the

same lattice set-up, using the HISQ (Highly Improved Staggered Quarks) action with tree-level Symanzik gauge action. We have mainly focused on the low-lying fermionic bound states (plus some gluonic ones), i.e., pseudoscalar (denoted as π), scalar (σ, a_0), vector (ρ), axialvector mesons (a_1), and nucleon-like states (N, N^*), etc. and particularly the flavor-singlet scalar σ as a candidate for the technidilaton.

We found [68] that $N_F = 12$ is consistent with the conformal window indicating no spontaneous chiral symmetry breaking, satisfying the universal hyperscaling relation $M \sim m_f^{1/(1+\gamma_m)}$ for all the observed quantities (with $\gamma_m \sim 0.4 \ll 1$), which is in agreement with the results of many other groups [72].

We further found [69] that $N_F = 8$, in comparison with $N_F = 12$ and $N_F = 4$ (up-dated in [76]), is consistent with the SSB phase with remnants of the conformality (approximately universal hyperscaling relation *except for the NG boson pion mass*) with a large anomalous dimension

$$\gamma_m \simeq 1, \quad (89)$$

namely the walking theory, which was confirmed by other groups [73].

A remarkable result of the LatKMI Collaboration is the *discovery of a light flavor-singlet scalar on the lattice*, lighter than the pion in $N_F = 12$ [70]. This $N_F = 12$ result of us was confirmed by other groups [74]. Since the theory is consistent with the conformal window without SSB, such a light flavor-singlet scalar in $N_F = 12$ may have no direct relevance to the technidilaton as a composite Higgs. Nevertheless, it is suggestive that the conformal dynamics may produce the dilaton scalar, which was generated only by the explicit breaking fermion bare mass m_0 put on the lattice.

Furthermore, we made an outstanding discovery that in $N_F = 8$ there exists a light flavor-singlet scalar with mass comparable to the pion [71]. See Fig. 3. Our $N_F = 8$ results were also

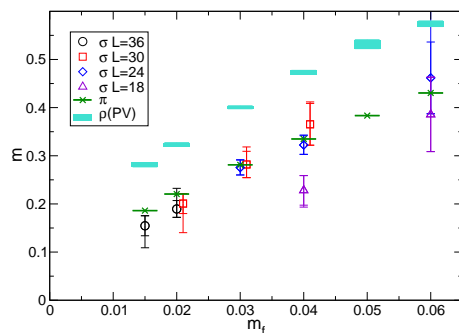


Fig. 3 Flavor-singlet scalar meson denoted as σ in $N_f = 8$ QCD HISQ with $\beta = 6/g^2 = 3.8$, in comparison with the NG boson pion π and the vector meson ρ , for various values of the degenerate fermion bare mass m_f on the lattice. Lattice volumes are $L^3 \times T = 36^3 \times 48, 30^3 \times 40, 24^3 \times 32, 18^3 \times 24$.

confirmed by other group [75]. Since $N_F = 8$ seems to be a walking theory in the SSB phase as mentioned above, the light flavor-singlet scalar is particularly attractive as a candidate for the technidilaton. Also $N_F = 8$ is of phenomenological relevance to the LHC data and of direct relevance to the one-family model as the most natural model building. Future

confirmation of our results is highly desired. Also $N_C = 4$ simulations should be studied for various reasons as mentioned before.

9. Conclusion

We have discussed the modern version of the Nambu's path to the origin of mass, namely the dynamical symmetry breaking of the electroweak symmetry, which may account for the origin of the otherwise mysterious input mass parameter (tachyon mass) of the SM Higgs Lagrangian. The dynamics changes the vacuum by the strong coupling attractive forces so as to produce the mass scale $m_F (\ll \Lambda)$ smaller than the intrinsic scale $\Lambda (\Lambda_{\text{QCD}})$ which is given either at Lagrangian level (like NJL model) or induced as the trace anomaly when regularizing the quantum theory (like QCD and technicolor). Crucial to the hierarchy $m_F \ll \Lambda$ is the non-zero critical coupling, which yields a large anomalous dimension and infrared conformality even in the NJL type four-fermion theory having the coupling of explicit mass scale.

We have defined “strong coupling theories” as “those having non-zero critical coupling” $N_C g_{\text{cr}} = \mathcal{O}(1)$, even though its value could be small $g \sim 1/N_C \ll 1$ in the typical large N_C limit. The NJL model pioneered by Professor Nambu is the first and a typical example of such, to be distinguished from its preceding, the BCS theory, which has a zero critical coupling $g_{\text{cr}} = 0$. Existence of such a non-zero critical coupling in gauge theory was discovered by Maskawa and Nakajima in the scale-invariant dynamics, ladder approximation, and became crucial for the walking technicolor with the coupling $N_C \alpha > N_C \alpha_{\text{cr}} = \mathcal{O}(1)$ in the SSB phase of the scale symmetry as well as the chiral symmetry.

The existence of the non-zero critical coupling is actually “hidden” even in the QCD which is regarded to have only one phase in the ordinary situation without signal of the non-zero critical coupling: it manifests itself in the extreme condition, such as the large number of fermions $N_F \gg N_C$ (so as to keep the asymptotic freedom), high temperature, high density, etc..

Indeed, it is the large N_F QCD that models the walking technicolor where the large number of fermions give the screening effects and level off of the infrared coupling which otherwise blows up due to the gluon anti-screening effects (Caswell-Banks-Zaks infrared fixed point). For large N_F with the fixed point value smaller than the critical coupling, the SSB phase is gone (what we called conformal phase transition). Then the infrared scale invariance becomes manifest, dubbed the conformal window. The walking technicolor is close to just outside of the conformal window. Although N_f and N_C are integers, the anti-Veneziano limit makes the analyses of the phase in the almost continuous parameter $r = N_F/N_C$.

To see the relevance of the infrared conformality, we have argued that the 125 GeV Higgs itself is a (pseudo-)dilaton, with mass coming from the trace anomaly of dimension 2 due to the Lagrangian parameter, even if it is described by the Standard Model Higgs Lagrangian (!!). The SM Higgs Lagrangian was in fact shown to be equivalent to the scale-invariant nonlinear sigma model with both chiral and scale symmetries being nonlinearly realized. The SM Higgs Lagrangian was further shown to be gauge equivalent to the scale-invariant Hidden Local Symmetry (HLS) Lagrangian which include new massive vector bosons as the gauge bosons of the (spontaneously broken) HLS, with the mass term being scale-invariant.

All these features of the SM Higgs Lagrangian are reminiscent of the conformal UV completion behind the Higgs, the underlying theory with (approximate) scale symmetry with

the coupling so strong as to produce composite states such as the Higgs (dilaton), new vector bosons (HLS gauge bosons), etc.. We have seen that even the NJL model, though not gauge theory, can be regarded as such a conformal UV completion.

The walking technicolor, conformal SCGT, is a gauge theory version of such a typical underlying theory, where the 125 GeV Higgs is a composite pseudo-dilaton, technidilaton, with mass coming from the nonperturbative trace anomaly. The walking technicolor in the anti-Veneziano limit $N_C \rightarrow \infty$ with $N_C \alpha = \text{fixed} = \mathcal{O}(1)$ and $N_F/N_C = \text{fixed} (\gg 1)$ makes the ladder approximation reasonable, which yields a naturally light and weakly coupled technidilaton through the PCDC:

$$M_\phi^2 F_\phi^2 = -4\langle\theta_\mu^\mu\rangle = -\frac{\beta(\alpha(\mu^2))}{\alpha(\mu^2)} \langle G_{\mu\nu}^2(\mu^2) \rangle \simeq N_C N_F \frac{16}{\pi^4} m_F^4, \quad (90)$$

independently of the renormalization point μ , where the scale symmetry is explicitly broken by the nonperturbative trace anomaly of the dimension 4 operator $G_{\mu\nu}^2$, which is induced by m_F the dynamical mass of the technifermion arising from the simultaneous spontaneous breaking of the scale symmetry and the chiral symmetry. The technidilaton with mass 125 GeV and coupling by the PCDC relation for the one-family model with $N_F = 8, N_c = 4$ is consistent with the present data for the LHC 125 GeV Higgs.

Lattice results are also encouraging for the light composite Higgs as the technidilaton in the walking technicolor, particularly for $N_F = 8$ QCD, which correspond to the one-family technicolor as the most straightforward walking technicolor model building.

We will see the fate of the strong coupling theories at LHC Run II, hoping that the Nambu's way will continue forever.

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